Natural Logs

Compound Interest formula for the amount A

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

As n gets very large, the interest is begins to be compounded continuously... all of the time without taking a break. When interest is compounded continuously, the formula above can be simplified using the natural base e. What in the world is the value of the number e?

$e \approx 2.71828...$

$2^{nd} \rightarrow \div$

Compounded Continuously

$$A = Pe^{rt}$$

Resulting amount

rate (decimal)

# of years

Principal (starting amount)

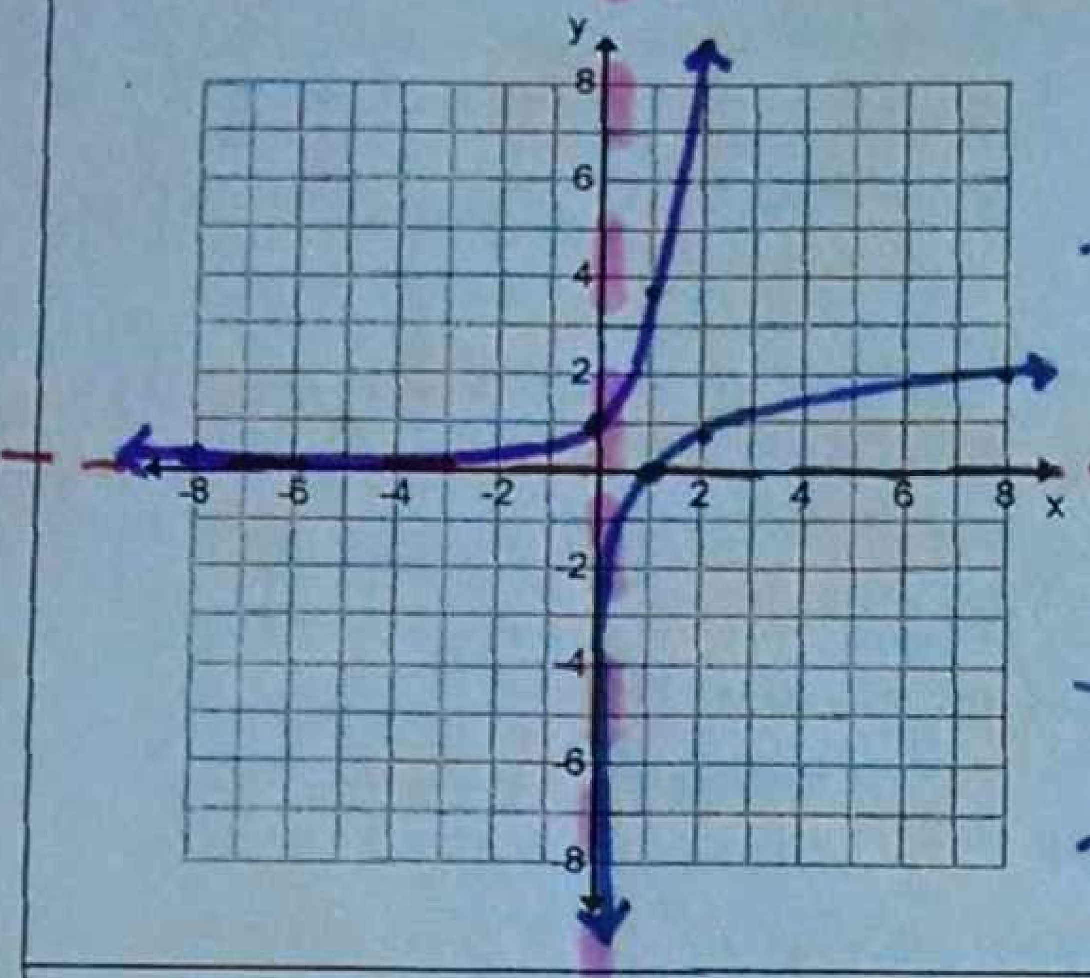
A logarithm with a base of e is called a natural logarithm and is abbreviated as "ln" (rather than $\log_e$). Natural logarithms have the same properties as log base 10 and logarithms with other bases.

The natural logarithmic function $f(x) = \ln(x)$ is the inverse of the natural exponential function $g(x) = e^x$. All of the properties from the previous unit all apply (Product, Quotient and Power)

Graphs of Exponential and Logarithmic Functions

x=0 vertical asymptote

$f(x) = e^x$ | $g(x) = \ln(x)$



D: ℝ

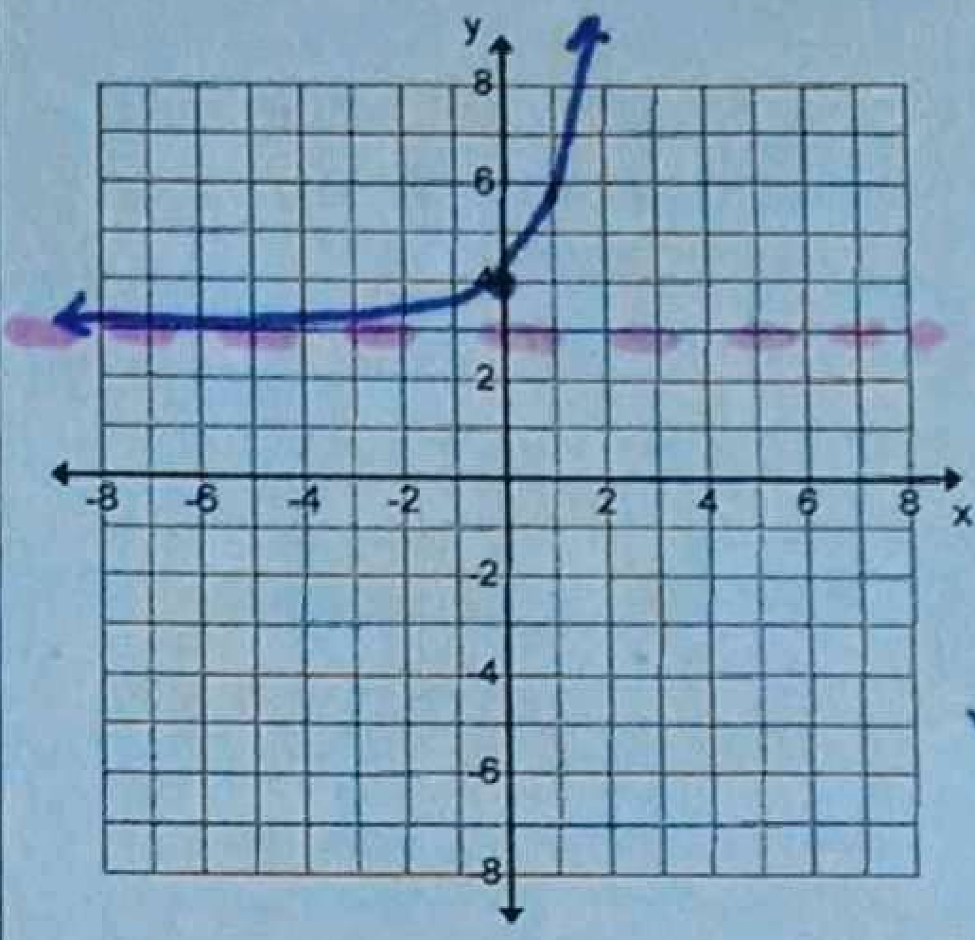
R: y > 0

-> horizontal asymptote y=0

D: x > 0

R: ℝ

$h(x) = e^x + 3$



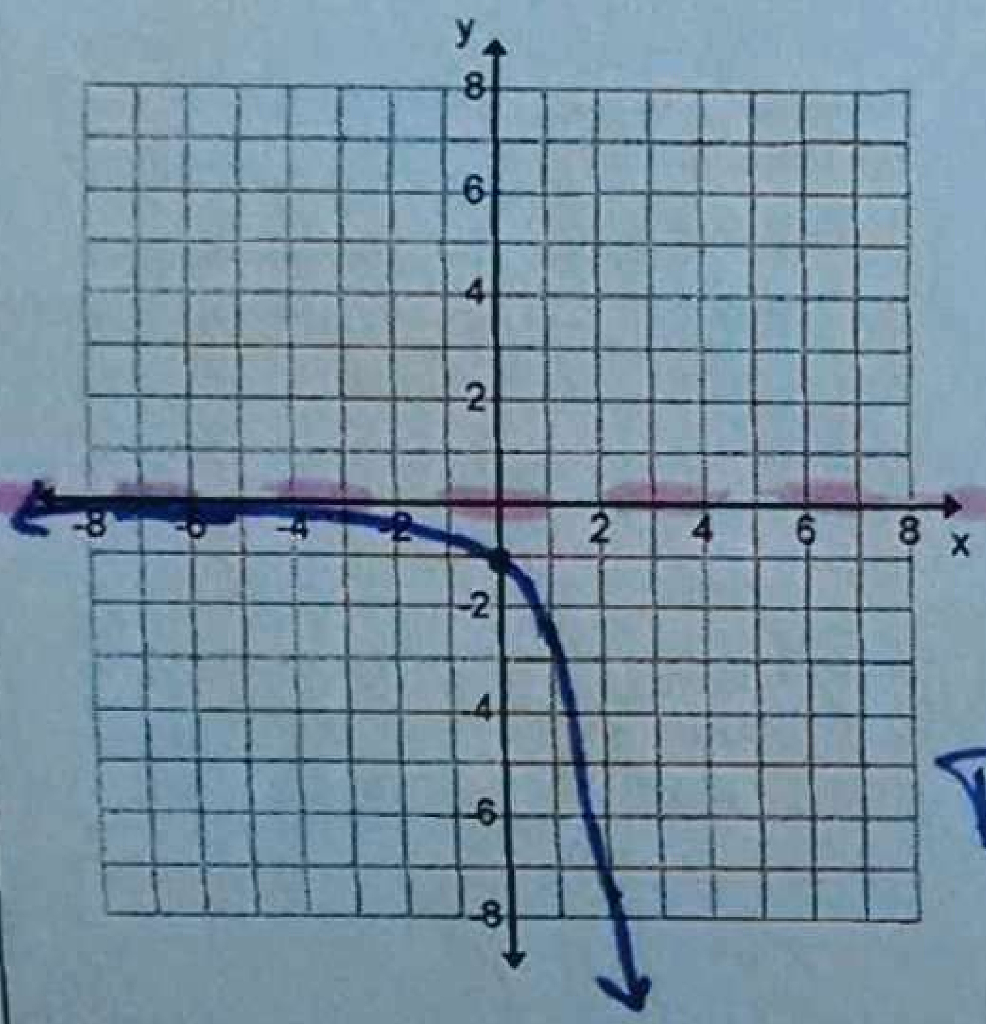
D: ℝ

R: y > 3

Horiz Asymptote y = 3

Vertical shift ↑ 3

$y = -e^x$



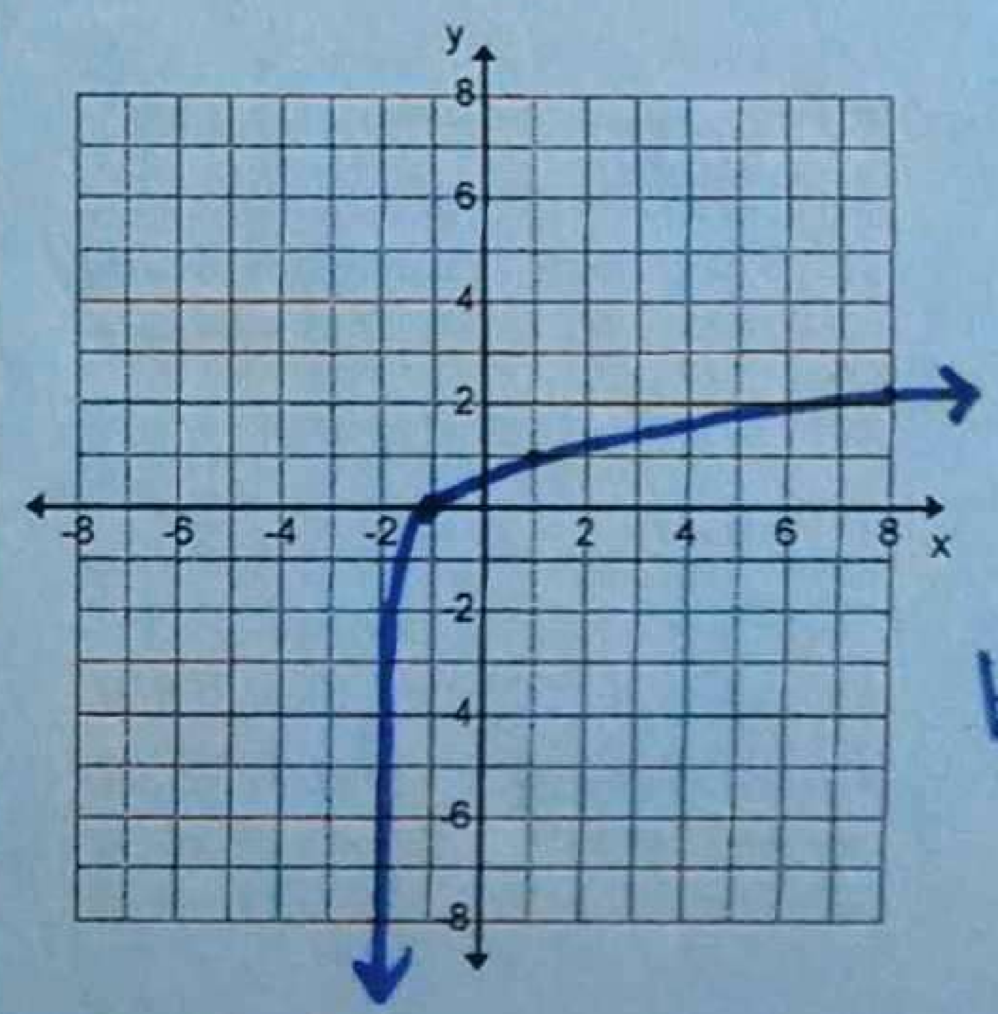
D: ℝ

R: y < 0

y = 0

Reflect across x-axis

$p(x) = \ln(x + 2)$



D: x > -2

R: ℝ

Horizontal shift left 2

Applications of Natural Base $A = Pe^{rt}$

a. What is the total amount for an investment of \$1000 compounded at 5% for 10 years compounded continuously?	b. What is the total amount for an investment of \$4000 invested at 3.5% for 8 years and compounded continuously?
P = 1000 r = 5% → .05 t = 10 years $A = 1000e^{(.05)(10)}$ A = \$1,648.72	P = 4000 r = 3.5% → .035 t = 8 $A = 4000e^{(.035)(8)}$ A = \$5,292.52

Simplify and Round to the Nearest Hundredth

Power Quotient

a. $3\ln(10) - \ln(8) = 2x$ $\ln_e\left(\frac{10^3}{8}\right) = 2x$ $4.828 = 2x$ $\boxed{x = 2.41}$	b. $2\ln(5x) = \ln(x) + 1$ $2\ln(5x)^2 - \ln(x) = 1$ $\ln\left(\frac{25x^2}{x}\right) = 1$ $\ln(25x) = 1$ $e^1 = 25x \Rightarrow \frac{e}{25} \approx \boxed{.11}$	c. $\ln(3x - 2) = 5$ $e^5 = 3x - 2$ $\frac{e^5 + 2}{3} = \frac{3x}{3}$ $x \approx 50.14$
d. $\ln 4 - \ln x = 1$ $\ln\left(\frac{4}{x}\right) = 1$ $x \cdot e^1 = \frac{4}{\cancel{x}} \cdot \cancel{x}$ $\frac{ex}{e} = \frac{4}{e}$ $x = 1.47$	e. $\ln(5) + 2\ln(x) = 7$ $\ln_e(5x^2) = 7$ $\frac{e^7}{5} = \frac{5x^2}{5}$	f. $\ln(x) = 2 - \ln(x - 3)$