Solving Systems of Equations

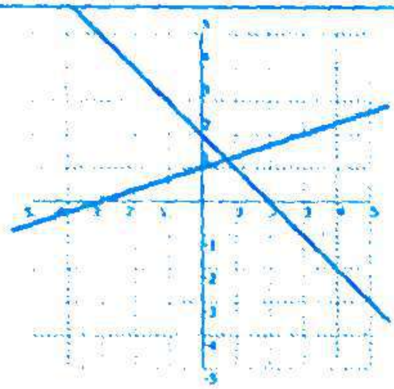
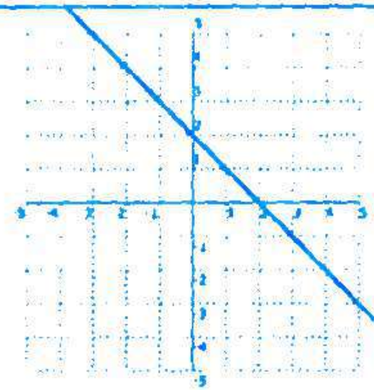
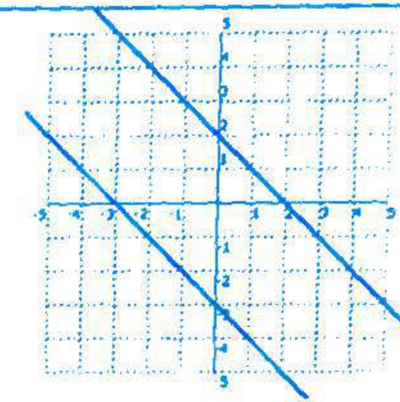
On a graph of the system of two equations, the solution is the set of all points where the lines intersect.

Determine if the given point is a solution to the system. (Substitute point into system)

a. $(2,4)$; $\begin{cases} x-2y=-6 \\ 2x+y=8 \end{cases}$ x y $2-2(4)=-6$ $2-8=-6$ $-6=-6$ ✓ $2(2)+4=8$ $4+4=8$ $8=8$ ✓	b. $(3,2)$; $\begin{cases} 2x+3y=12 \\ 8x-6y=24 \end{cases}$ x y $2(3)+3(2)=12$ $6+6=12$ $12=12$ ✓ $8(3)-6(2)=24$ $24-12=24$ $12\neq 24$ X	c. $(5,3)$; $\begin{cases} 6x-7y=1 \\ 3x+7y=5 \end{cases}$ x y $6(5)-7(3)=1$ $30-21=1$ $9\neq 1$ X Not a Solution.

(2,4) is a Solution

(3,2) is not a solution

Classifying Linear Systems		
Exactly One Solution	**Infinitely Many Solutions**	**No Solution**
		
The graphs are intersecting lines with $\boxed{\text{different slopes.}}$	The graphs are coinciding lines; they have the $\boxed{\text{same slope and}}$ the $\boxed{\text{same } y\text{-intercept.}}$	The graphs are parallel lines; they have the $\boxed{\text{same slope}}$ but $\boxed{\text{different}}$ $\boxed{y\text{-intercepts.}}$

Example 3: Classify each system and determine the number of solutions. (Solve for y)

a. $\begin{cases} 2x + y = 3 \\ 6x = 9 - 3y \end{cases}$

$2x + y = 3$
$-2x \qquad -2x$
$\boxed{y = -2x + 3}$

$6x = 9 - 3y$
$6x - 9 = -3y$
$\boxed{y = -2x + 3}$

Same slope and same y-intercept
→ Coinciding Lines → Infinitely many Solutions

b. $\begin{cases} x + 4 = y \\ 5y = 5x + 35 \end{cases}$

$x + 4 = y$
$\boxed{y = x + 4}$

$5y = 5x + 35$
$\boxed{y = x + 7}$

Same slope but different y-intercepts
→ Parallel Lines → No Solution

By Graphing...

You can use your graphing calculator to graph a system of equations and then to find the solution. How do we get the calculator to find the point of intersection? Once you have the system graphed, press the keys:

2nd TRACE 5 ENTER ENTER ENTER

Equations must be in slope intercept for in order to enter it into calculator

Example 2: Use a graph and a table on your calculator to solve each system. Check your answer.

a.	b.	c.
$\begin{cases} x+y=4 \\ 2y+4=x \end{cases}$	$\begin{cases} 3x-y=-2 \\ x-y=-4 \end{cases}$	$\begin{cases} y-x=5 \\ 3x+y=1 \end{cases}$
$x+y=4$ $-x \quad -x$ $\boxed{y=-x+4}$ $2y+4=x$ $\frac{2y}{2}=\frac{x}{2}-\frac{4}{2}$ $\boxed{y=\frac{1}{2}x-2}$ $(4,0)$	$3x-y=-2$ $-y=-3x-2$ $\boxed{y=3x+2}$ $x-y=-4$ $-y=-x-4$ $\boxed{y=x+4}$ $(1,5)$	$y-x=5$ $\boxed{y=x+5}$ $3x+y=1$ $\boxed{y=-3x+1}$ $(-1,4)$

Solving Systems by Substitution

★ Use when one variable is solved for (isolated) or when one is easy to solve for

$$\begin{cases} y = 6x - 11 \\ -2x - 3y = -7 \end{cases}$$

$-2x - 3(6x - 11) = -7$

$-2x - 18x + 33 = -7$

$-20x + 33 = -7$

$-33 \quad -33$

$-20x = -40$

$\boxed{x = 2}$

Substitute 2 in for x

$y = 6x - 11$

$y = 6(2) - 11$

$y = 12 - 11$

$\boxed{y = 1}$

Solution $(2, 1)$

check:

$-2(2) - 3(1) = -7$

$-4 - 3 = -7$

$-7 = -7$ ✓

a. $\begin{cases} y = x + 2 \\ x + y = 8 \end{cases}$

$(3, 5)$

$x + (x + 2) = 8$

$2x + 2 = 8$

$2x = 6$

$\boxed{x = 3}$

$y = x + 2$

$y = 3 + 2$

$\boxed{y = 5}$

check:

$3 + 5 = 8$

$8 = 8$ ✓

$5 = 3 + 2$

$5 = 5$ ✓

b. $\begin{cases} 2x + y = 6 \\ y - 8x = 1 \rightarrow y = 8x + 1 \end{cases}$

$2x + (8x + 1) = 6$

$10x + 1 = 6$

$10x = 5$

$\boxed{x = \frac{1}{2}}$

$(\frac{1}{2}, 5)$

$y - 8(\frac{1}{2}) = 1$

$y - 4 = 1$

$\boxed{y = 5}$

c. $\begin{cases} 5x + 6y = -9 \\ 2x - 2 = -y \rightarrow y = -2x + 2 \end{cases}$

$5x + 6(-2x + 2) = -9$

$5x - 12x + 12 = -9$

$-7x + 12 = -9$

$-7x = -21$

$\boxed{x = 3}$

$2x - 2 = -y$

$2(3) - 2 = -y$

$6 - 2 = -y$

$4 = -y$

$\boxed{y = 4}$

$(3, -4)$

Solve by Elimination ★ Eliminate one of the variables by adding Equations

Same coefficient Opposite Sign	same coefficient same sign	Different Coefficients
· Add equations	· multiply one equation by -1, then add equations	· multiply one or both equations to get Least Common multiple w/ opposite signs, then add equations TOGETHER
a. $\begin{cases} 2x + 3y = 34 \\ 4x - 3y = -4 \end{cases}$ (+) $6x = 30$ $\boxed{x = 5}$ $2(5) + 3y = 34$ $10 + 3y = 34$ $3y = 24$ $\boxed{y = 8}$ $(5, 8)$	b. $\begin{cases} (8x + 2y = 30) -1 \\ 7x + 2y = 24 \end{cases}$ $-8x - 2y = -30$ $-x = -6$ $\boxed{x = 6}$ $7(6) + 2y = 24$ $42 + 2y = 24$ $2y = -18$ $\boxed{y = -9}$ $(6, -9)$	c. $\begin{cases} (3x - 2y = 2)\ 5 \\ (5x - 5y = 10)\ -3 \end{cases}$ $15x - 10y = 10$ $+ -15x + 15y = -30$ $5y = -20$ $\boxed{y = -4}$ $3x - 2(-4) = 2$ $3x + 8 = 2$ $3x = -6$ $\boxed{x = -2}$ $(-2, -4)$