

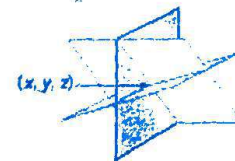
**GLUE HERE**

$$\begin{cases} -4x - y + 3z = -9 \\ 3x + 2y - z = 3 \\ x + 3y - 5z = 29 \end{cases}$$

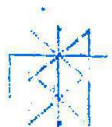


**DON'T BE UPSET  
 BY THE RESULTS  
 YOU DIDN'T GET  
 WITH THE WORK  
 YOU DIDN'T DO.**

## Solving Systems of Equations with 3 Equations and 3 Variables

The solution to a system of three variables can have one solution, which is called an ordered triple  $(x, y, z)$ , where three individual planes intersect at a point



It can also have...

Infinitely Many Solutions	No Solution
<p>The planes intersect in a line or planes intersect on the same plane</p>  <p>Or</p> 	<p>There are no points in common with all three planes.</p> 

Is the point  $(3, -2, -4)$  a solution to the system

$$\begin{cases} 2x + 4y - 5z = 18 \\ -3x + 5y + 2z = -27 \\ -5x + 3y - z = -17 \end{cases}$$

$$2(3) + 4(-2) - 5(-4) = 18 \checkmark$$

$$-3(3) + 5(-2) + 2(-4) = -27 \checkmark$$

$$-5(3) + 3(-2) - (-4) = -17 \checkmark$$

$(3, -2, -4)$  is a solution

Example 1: Find the Solution to the system.

$$\begin{cases} 3x - 2y + 4z = 35 \\ -4x + y - 5z = -36 \\ 5x - 3y + 3z = 31 \end{cases}$$

1) Eliminate 1 variable by using 2 pairs of 2 equations.

$$\begin{array}{r} 3x - 2y + 4z = 35 \\ (-4x + y - 5z = -36) \cdot 2 \\ \hline -8x + 2y - 10z = -72 \\ + 3x - 2y + 4z = 35 \\ \hline -5x - 6z = -37 \end{array}$$

$$\begin{array}{r} (-4x + y - 5z = -36) \cdot 3 \\ \hline -12x + 3y - 15z = -108 \\ + 5x - 3y + 3z = 31 \\ \hline -7x - 12z = -77 \end{array}$$

make sure you eliminate the same variable

2) Solve the system of 2 equations you just created.

$$\begin{array}{r} (-5x - 6z = -37) \cdot 2 \\ \hline -10x - 12z = -74 \\ + 7x + 12z = -77 \\ \hline -3x = -3 \\ \hline x = -1 \end{array}$$

$$\begin{array}{r} -5(-1) - 6z = -37 \\ 5 - 6z = -37 \\ -5 \quad -5 \\ \hline -6z = -42 \\ \hline z = 7 \end{array}$$

3) Substitute the values back into an original equation to find the 3<sup>rd</sup> value.

$$\begin{aligned} -4x + y - 5z &= -36 \\ -4(-1) + y - 5(7) &= -36 \\ 4 + y - 35 &= -36 \\ y - 31 &= -36 \\ +31 \quad +31 & \\ \hline y &= -5 \end{aligned}$$

$(-1, -5, 7)$

$$\begin{cases} 2x + 4y - 5z = 18 \\ -3x + 5y + 2z = -27 \\ -5x + 3y - z = -17 \end{cases}$$

$(3, -2, 4)$

$$\begin{array}{r} 2x + 4y - 5z = 18 \\ (-5x + 3y - z = -17) \cdot 2 \\ \hline -10x + 6y - 2z = -34 \\ + 2x + 4y - 5z = 18 \\ \hline -8x + 10y - 7z = -16 \end{array}$$

$$\begin{array}{r} (-5x + 3y - z = -17) \cdot 2 \\ \hline -10x + 6y - 2z = -34 \\ + 2x + 4y - 5z = 18 \\ \hline -8x + 10y - 7z = -16 \end{array}$$

$$\begin{array}{r} 27x - 11y = 103 \\ + (-13x + 11y = -61) \\ \hline 14x = 42 \\ \hline x = 3 \end{array}$$

$$\begin{array}{r} -13(3) + 11y = -61 \\ -39 + 11y = -61 \\ 11y = -22 \\ \hline y = -2 \end{array}$$

Special Systems

$$\begin{cases} 5x + 4y - 5z = -10 \\ -4x - 10y - 8z = -16 \\ 6x + 15y + 12z = 24 \end{cases}$$

$$\begin{array}{r} (5x + 4y - 5z = -10) \cdot 4 \\ \hline 20x + 16y - 20z = -40 \\ + (-4x - 10y - 8z = -16) \cdot 5 \\ \hline 20x + 16y - 20z = -40 \\ + (-20x - 50y - 40z = -80) \\ \hline -34y - 60z = -120 \end{array}$$

$$\begin{array}{r} (-4x - 10y - 8z = -16) \cdot 6 \\ \hline -24x - 60y - 48z = -96 \\ + (6x + 15y + 12z = 24) \cdot 4 \\ \hline -24x - 60y - 48z = -96 \\ + (24x + 60y + 48z = 96) \\ \hline 0 = 0 \end{array}$$

$$2x + 4y - 5z = 18$$

$$2(3) + 4(-2) - 5z = 18 \quad z = 4$$

$$\begin{cases} -6a + 9b - 12c = 21 \\ -2a + 3b - 4c = 7 \\ 10a - 15b + 20c = -30 \end{cases}$$

$$\begin{array}{r} -6a + 9b - 12c = 21 \\ (-2a + 3b - 4c = 7) \cdot 3 \\ \hline -6a + 9b - 12c = 21 \\ + (-6a + 9b - 12c = 21) \\ \hline -12a + 18b - 24c = 42 \\ + (10a - 15b + 20c = -30) \\ \hline -2a + 3b - 4c = 12 \end{array}$$

$$0 = 0$$

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