Linear programming is a method of finding a maximum or minimum value of a function that satisfies a given set of conditions called constraints/restrictions. A constraint is one of the inequalities in a linear programming problem. The solution to the set of constraints can be graphed as a feasible region (shading overlaps)

Example 1: Graph the feasible region for the given constraints. Then find the vertices of the region.

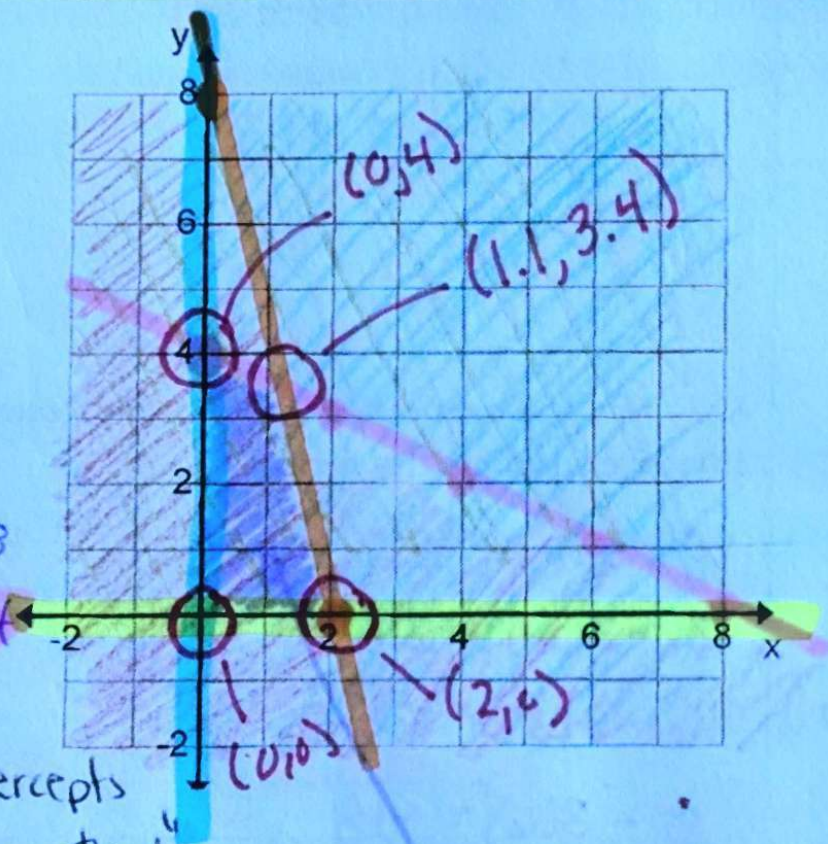
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \leq 8 \\ 4x + y \leq 8 \end{cases}$$

Solve for y

$x + 2y \leq 8$

$2y \leq -x + 8$

$y \leq -\frac{1}{2}x + 4$



Feasible region

Find intercepts "cover up method"

$4x + y \leq 8$

$x = 2$ $y = 8$

$(2,0)$ $(0,8)$

Objective Function

Maximize $\boxed{P = 2x + 3y}$ for the feasible region

$(0,4)$: $2(0) + 3(4) = 12$

$(0,0)$: $2(0) + 3(0) = 0$

$(2,0)$: $2(2) + 3(0) = 4$

$(1.1,3.4)$: $2(1.1) + 3(3.4) = 12.4$

Minimize $\boxed{C = 2x + y}$ for the feasible region

$(0,4)$: $2(0) + 4 = 4$

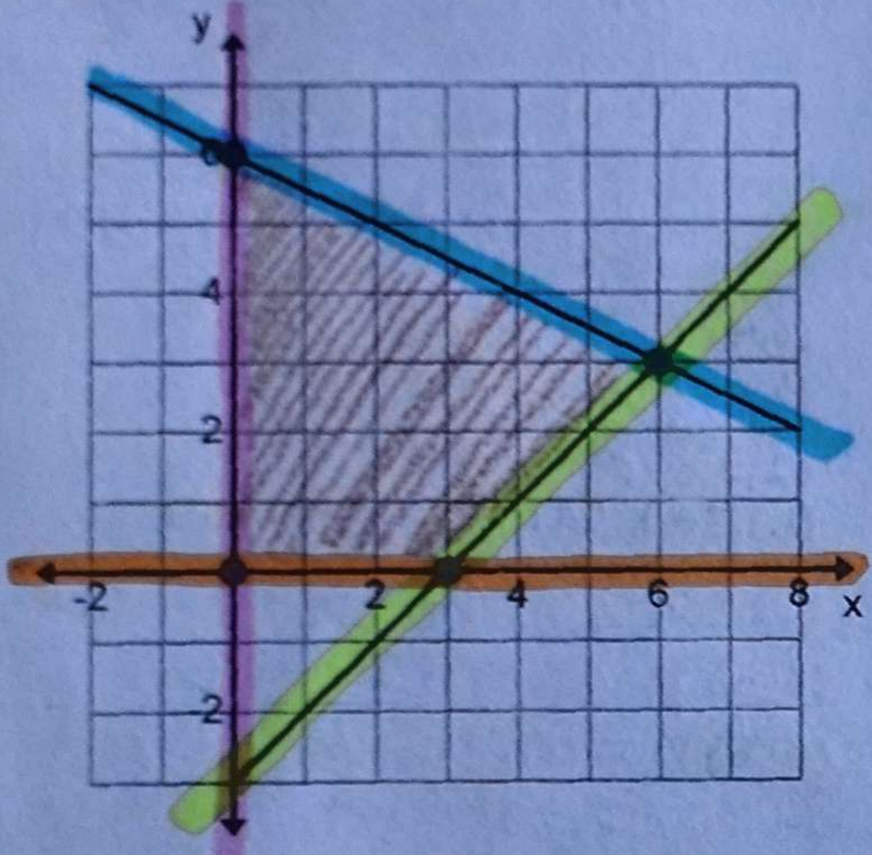
$(0,0)$: $2(0) + 0 = 0$

$(2,0)$: $2(2) + 0 = 4$

$(1.1,3.4)$: $2(1.1) + 3.4 = 5.6$

Minimum value of 0 @ (0,0)

Given the graph of the feasible region, identify the figure and write the inequalities that represent the constraints.



$(0,0)$ $(3,0)$ $(6,3)$ $(0,6)$

$x \geq 0$

$y \geq 0$

$y \geq x - 3$

$y \leq -\frac{1}{2}x + 6$

Now, find the vertices of the feasible region and maximize the objective function $P = x - 3y$.

$0 - 3(0) = 0$

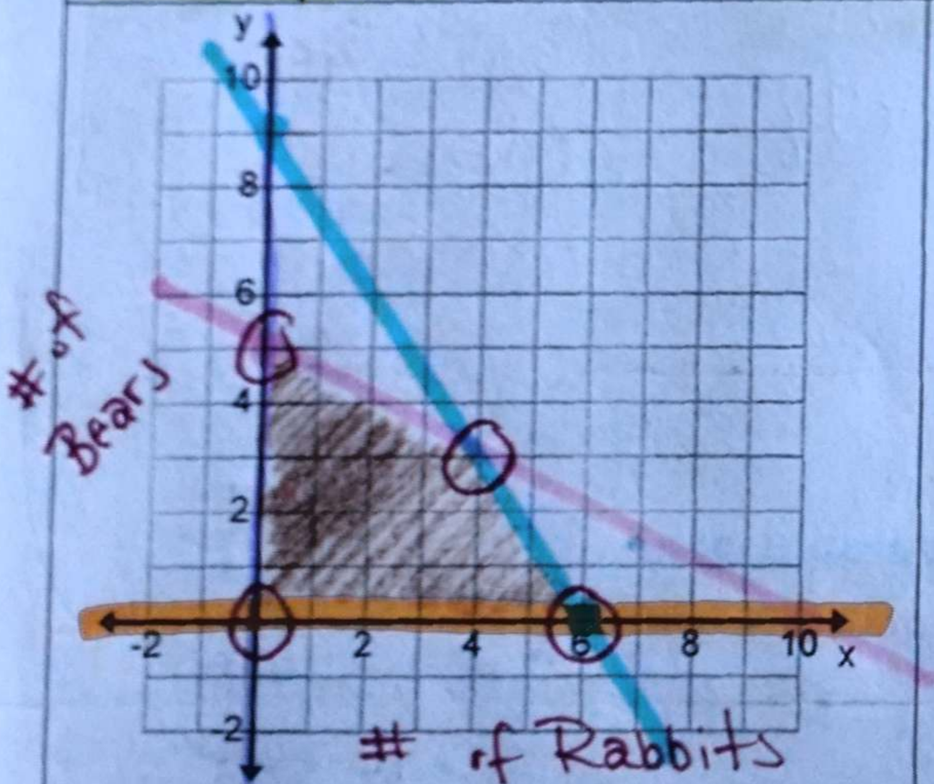
$3 - 3(0) = \boxed{3}$

$6 - 3(3) = -3$

$0 - 3(6) = -18$

Max value of 3 @ (3,0)

Amy spends at most 40 hours per week making stuffed rabbits and bears. Four hours are required to make a rabbit and 8 hours to make a bear. The material for each rabbit costs \$15 and the material for each bear costs \$10. Amy can afford to spend at most \$90 per week on materials. The profit on one rabbit is \$8 and the profit on one bear is \$11. How many rabbits and how many bears should Amy sell per week in order to obtain the maximum profit?



Objective Function:

$P = 8x + 11y$

Constraints:

$15x + 10y \le 90$

$4x + 8y \le 40$

$x \ge 0$

$y \ge 0$

(0,5) (6,0)

(4,3) (0,0)

$P = 8x + 11y$

$8(0) + 11(5) = 55$

$8(6) + 11(0) = 48$

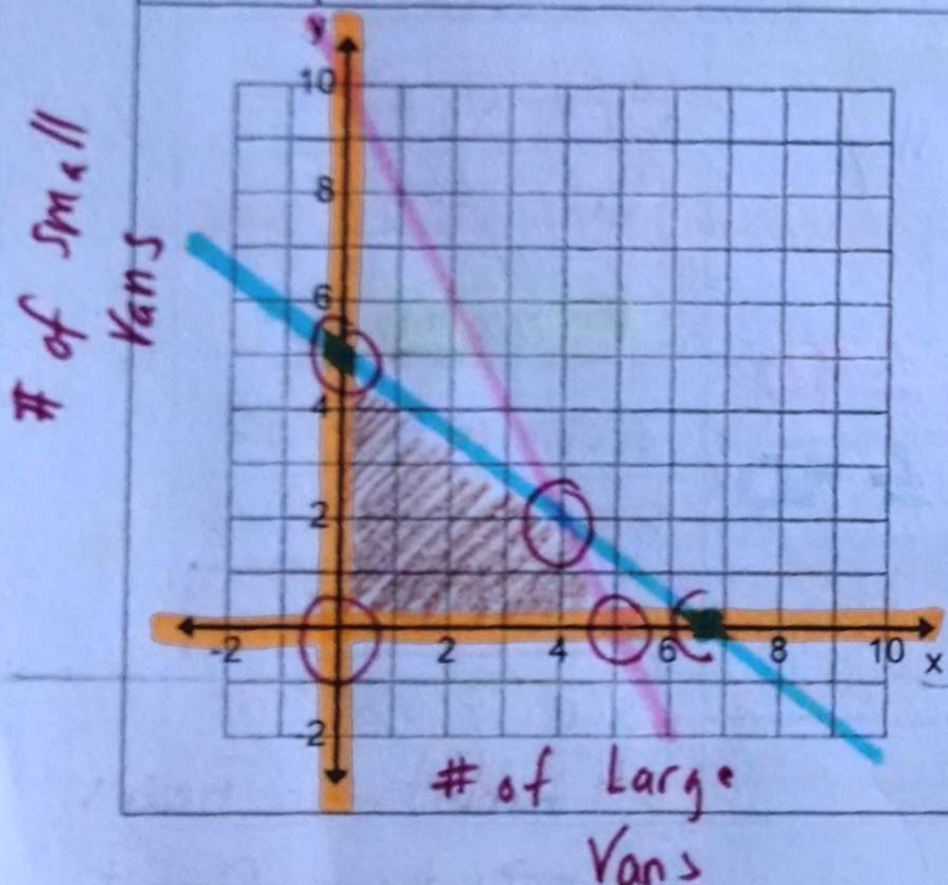
$\boxed{8(4) + 11(3) = 65}$

$8(0) + 6(0) = 0$

4 Rabbits + 3 Bears

$\Rightarrow$ \$65 Profit

Buda is trying to establish a public transportation system of large and small vans. It can spend no more than$100,000 for both sizes of vans and no more than $500 per month to maintain the vans. A small van costs $10,000 and$100 per month to maintain it. A large van costs $20,000 and costs $75 per month to maintain. The large van carries a max of 15 passengers and the small on has a max of 7 passengers. How many small and large vans can be purchased to accommodate the maximum number of passengers?



Objective Function:

$P = 15x + 7y$

Constraints:

$x \geq 0$

$y \geq 0$

$20{,}000x + 10{,}000y \leq 100000$

$75x + 100y \leq 500$

(0,0) (5,0)

(0,5) (4,2)

(0,0) = 0

(0,5) = 35

(4,2) = 74

$\boxed{(5,0) = 75}$

5 large vans

⇒ 75 passengers