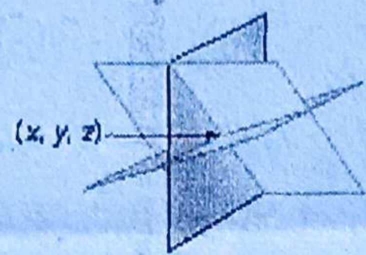
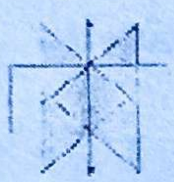
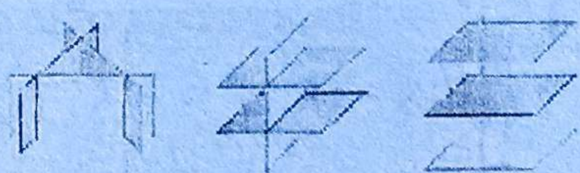
Solving Systems of Equations with 3 Equations and 3 Variables

The solution to a system of three variables can have one solution, which is called an ordered triple (x, y, z), where three individual planes intersect at a s



It can also have…

Infinitely Many Solutions	No Solution
The planes intersect in a line or planes intersect on the same plane  Or 	There are no points in common with all three planes. 

Is the point (3, -2, -4) a solution to the system

$$\begin{aligned} 2x + 4y - 5z &= 18 \\ -3x + 5y + 2z &= -27 \\ -5x + 3y - z &= -17 \end{aligned}$$?

x y z

$2(3) + 4(-2) - 5(-4) = 18$ ✓

$-3(3) + 5(-2) + 2(-4) = -27$ ✓

$-5(3) + 3(-2) - (-4) = -17$ ✓

(3, -2, -4) is a solution

Example 1: Find the Solution to the system.

$3x - 2y + 4z = 35$

$-4x + y - 5z = -36$

$5x - 3y + 3z = 31$

1) Eliminate 1 variable by using 2 pairs of 2 equations.

$3x - 2y + 4z = 35$

$(-4x + y - 5z = -36)2$

$-8x + 2y - 10z = -72$

$+ \; 3x - 2y + 4z = 35$

$-5x - 6z = -37$

$3(-4x + y - 5z = -36)$

$5x - 3y + 3z = 31$

$+ \; -12x + 3y - 15z = -108$

$-7x - 12z = -77$

Make sure you eliminate the same variable

2) Solve the system of 2 equations you just created.

$(-5x - 6z = -37)-2$

$-7x - 12z = -77$

$+ \; 10x + 12z = 74$

$3x = -3$

$x = -1$

$-5(-1) - 6z = -37$

$5 - 6z = -37$

$-5 \quad -5$

$-6z = -42$

$z = 7$

3) Substitute the values back into an original equation to find the 3rd value.

$-4x + y - 5z = -36$

$-4(-1) + y - 5(7) = -36$

$4 + y - 35 = -36$

$y - 31 = -36$

$y = -5$

$(-1, -5, 7)$

$(3, -2, -4)$

$2x + 4y - 5z = 18$

$-3x + 5y + 2z = -27$

$-5x + 3y - z = -17$

$2x + 4y - 5z = 18$

$(-5x + 3y - z = -17)-5$

$25x - 15y + 5z = 85$

$+ \; 2x + 4y - 5z = 18$

$27x - 11y = 103$

$-3x + 5y + 2z = -27$

$(-5x + 3y - z = -17)2$

$-10x + 6y - 2z = -34$

$+ \; -3x + 5y + 2z = -27$

$-13x + 11y = -61$

$-13x + 11y = -61$

$+ \; 27x - 11y = 103$

$14x = 42$

$x = 3$

$-13(3) + 11y = -61$

$-39 + 11y = -61$

$11y = -22$

$y = -2$

$2x + 4y - 5z = 18$

$2(3) + 4(-2) - 5z = 18$

$z = -4$

Special Systems

$5x + 4y - 5z = -10$

$-4x - 10y - 8z = -16$

$6x + 15y + 12z = 24$

$(-4x - 10y - 8z = -16)3$

$(6x + 15y + 12z = 24)2$

$-12x - 30y - 24z = -48$

$+ \; 12x + 30y + 24z = 48$

$0 = 0$

ℝ

$-6a + 9b - 12c = 21$

$-2a + 3b - 4c = 7$

$10a - 15b + 20c = -30$

$(-2a + 3b - 4c = 7)5$

$10a - 15b + 20c = -30$

$+ \; -10a + 15b - 20c = 35$

$0 = 5$

$\emptyset$