Inverse of a Matrix

When Matrix A is multiplied by its inverse, A^{-1}, the result is the identity matrix

$$\begin{bmatrix}1&0\\0&1\end{bmatrix} \qquad \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

- A matrix can have an inverse only if it is a square matrix but not all square matrices have an inverse
- To find the inverse of a matrix:

  * Find the determinant

  * If the matrix has an inverse use the formula below given for Matrix A:

$$A=\begin{bmatrix}a&b\\c&d\end{bmatrix} \qquad \boxed{A^{-1}=\frac{1}{\det A}\cdot\begin{bmatrix}d&-b\\-c&a\end{bmatrix}}$$

A+D switch places
B+C switch signs

Find the inverse of the matrix, if defined.

$$\begin{bmatrix}6&5\\8&3\end{bmatrix}^{-1}=\frac{1}{-22}\begin{bmatrix}3&-5\\-8&6\end{bmatrix}=\begin{bmatrix}-3/22&5/22\\4/11&-3/11\end{bmatrix}$$

$$\begin{bmatrix}\frac{1}{3}&\frac{2}{3}\\-6&3\end{bmatrix}^{-1}=\frac{1}{5}\begin{bmatrix}3&-2/3\\+6&1/3\end{bmatrix}=\begin{bmatrix}3/5&-2/15\\6/5&1/15\end{bmatrix}$$

$$\begin{bmatrix}0.2&30\\-0.3&5\end{bmatrix}^{-1}=\frac{1}{10}\begin{bmatrix}5&-30\\.3&.2\end{bmatrix}=\begin{bmatrix}1/2&-3\\.03&.02\end{bmatrix}$$

Determinant of a Matrix

Used to determine if a matrix has an inverse

- Only use with square matrices
- Determinant of a 2X2 matrix is the difference of the products of the diagonals

$$\det\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}a&b\\c&d\end{bmatrix}=(a)(d)-(b)(c)$$
$$=ad-bc$$

- If the determinant = 0, the matrix does not have an inverse
- If the determinant is not equal to 0, the matrix has an inverse

Find the determinant of each matrix.

$\begin{bmatrix}6&5\\8&3\end{bmatrix}$ $6(3)-5(8)=\boxed{-22}$ determinant
has an inverse

$\begin{bmatrix}\frac{1}{3}&\frac{2}{3}\\-6&3\end{bmatrix}$ $3(\frac{1}{3})-(-6)(\frac{2}{3})=5$
has an inverse

$\begin{bmatrix}6&8\\3&4\end{bmatrix}=0$ no inverse

$\begin{bmatrix}0.2&30\\-0.3&5\end{bmatrix}$ $5(.2)-30(-.3)=10$
has inverse

How to find the inverse of a matrix using the calculator?

Once you've entered the matrix into the calculator...

$\underbrace{[\text{2nd}]\ [x^{-1}]}_{\text{Matrix Menu}}\ \underset{\text{selection}}{[\text{ENTER}]}\ \overset{\text{Inverse}}{[x^{-1}]}\ [\text{ENTER}]$

$$\begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & -1 \\ 3 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1/4 & -1/8 & 3/8 \\ 1/2 & 3/4 & -1/4 \\ 3/4 & 3/8 & -1/8 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} -.75 & .875 \\ .25 & -.125 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}^{-1}$$ No Inverse $\Rightarrow$ Singular Matrix

det = 0

How to find the determinant using the calculator?

Once you've entered the matrix into the calculator...

$\underbrace{[\text{2nd}]\ [x^{-1}]}_{\text{Matrix Menu}}\ \underset{\text{Math}}{[\ )\ ]}\ \overset{\text{det}}{[1]}\ \underbrace{[\text{2nd}]\ [x^{-1}]\ [1]\ [\ )\ ]}_{\text{Select matrix}}\ [\text{ENTER}]$

$$\begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & -1 \\ 3 & 0 & 1 \end{bmatrix} \quad \det = -8$$

$$\begin{bmatrix} 8 & 2 \\ 4 & -1 \end{bmatrix} = -16$$

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} = -10$$

$$\begin{bmatrix} -6 & 3 \\ 9 & -5 \end{bmatrix} = 3$$

Using Inverse Matrices to Solve Systems

The matrix equation representing $\begin{cases} x+y=8 \\ 2x+y=1 \end{cases}$ is shown.

Matrix EQUATION

Matrix A
Coefficients of the System

$$\boxed{\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 8 \\ 1 \end{bmatrix}}$$

Matrix B
constants

Matrix X
(variables)

To solve $AX = B$, multiply both sides by the inverse A^{-1}.

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

Solution: (Inverse of A) • B

$Ax + By = C$

EQUATIONS MUST BE IN STANDARD FORM

1) $2x + 5y = 0$ ~~$3y = 5x - 31$~~ $-5x + 3y = -31$ $\underset{A}{\begin{bmatrix} 2 & 5 \\ -5 & 3 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 0 \\ -31 \end{bmatrix}}$ $A^{-1}B = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$	2) $13x - y = 8$ ~~$-2x = -4$~~ $-2x + 0y = -4$ $\begin{bmatrix} 13 & -1 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$ $x = \begin{bmatrix} 2 \\ 18 \end{bmatrix}$

$x=5 \quad y=-2$

a) $\begin{cases} x+y=8 \\ 2x+y=1 \end{cases}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} -7 \\ 15 \end{bmatrix}$$

b) $\begin{cases} y=-x+4 \\ 2x=9-3y \end{cases}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

c) $\begin{aligned} 3a+6b-2c&=-6 \\ 2a+b+4c&=19 \\ -5a-2b+8c&=62 \end{aligned}$

$$\begin{bmatrix} 3 & 6 & -2 \\ 2 & 1 & 4 \\ -5 & -2 & 8 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -6 \\ 19 \\ 62 \end{bmatrix}$$

$$X = \begin{bmatrix} -4 \\ 3 \\ 6 \end{bmatrix}$$

d) $\begin{aligned} x+2y&=12 \\ 3y-4z&=25 \\ x+6y+z&=20 \end{aligned}$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -4 \\ 1 & 6 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 25 \\ 20 \end{bmatrix}$$

$$X = \begin{bmatrix} 6 \\ 3 \\ -4 \end{bmatrix}$$

GLUE HERE