

Inverse of a Matrix

When Matrix A is multiplied by its inverse, A^{-1} , the result is the identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- A Matrix can have an inverse only if it is a square matrix but not all square matrices have an inverse
- To find the inverse of a matrix:
 - * Find the determinant
 - * If the matrix has an inverse use the formula below given for matrix A:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

A + D switch places
B + C switch signs

Determinant of a Matrix

Used to determine if a matrix has an inverse

- Only use with square matrices
- Determinant of a 2x2 matrix is the difference of the products of the diagonals

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{matrix} a & b \\ c & d \end{matrix} = (a)(d) - (b)(c) = ad - bc$$

- If the determinant = 0, the matrix does not have an inverse
- If the determinant is not equal to 0, the matrix has an inverse

Find the inverse of the matrix, if defined.

$$\begin{bmatrix} 6 & 5 \\ 8 & 3 \end{bmatrix}^{-1} = \frac{1}{-22} \begin{bmatrix} 3 & -5 \\ -8 & 6 \end{bmatrix} = \begin{bmatrix} -3/22 & 5/22 \\ 4/11 & -3/11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 3 \\ -6 & 3 \end{bmatrix}^{-1} = \frac{1}{5} \begin{bmatrix} 3 & -2/3 \\ +6 & 1/3 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/15 \\ 6/5 & 1/15 \end{bmatrix}$$

$$\begin{bmatrix} 0.2 & 30 \\ -0.3 & 5 \end{bmatrix}^{-1} = \frac{1}{10} \begin{bmatrix} 5 & -30 \\ .3 & .2 \end{bmatrix} = \begin{bmatrix} .5 & -3 \\ .03 & .02 \end{bmatrix}$$

Find the determinant of each matrix.

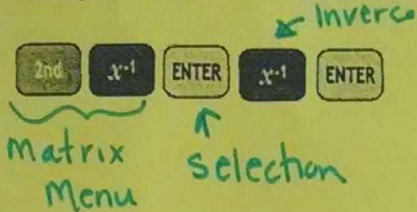
$$\begin{bmatrix} 6 & 5 \\ 8 & 3 \end{bmatrix} \quad 6(3) - 5(8) = -22 \quad \text{determinant} \\ \text{has an inverse}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 3 \\ -6 & 3 \end{bmatrix} \quad 3(\frac{1}{3}) - (-6)(\frac{2}{3}) = 5 \quad \begin{bmatrix} 6 & 8 \\ 3 & 4 \end{bmatrix} = 0 \\ \text{has an inverse} \quad \text{no inverse}$$

$$\begin{bmatrix} 0.2 & 30 \\ -0.3 & 5 \end{bmatrix} \quad 5(2) - 30(-.3) = 10 \\ \text{has inverse}$$

How to find the inverse of a matrix using the calculator?

Once you've entered the matrix into the calculator...



$$\begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & -1 \\ 3 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1/4 & -1/8 & 3/8 \\ 1/2 & 3/4 & -1/4 \\ 3/4 & 3/8 & -1/8 \end{bmatrix}$$

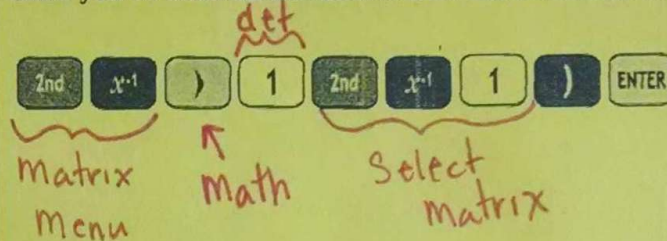
$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ -1/6 & 1/3 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} -.75 & .875 \\ .25 & -.125 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}^{-1} \text{ No Inverse } \Rightarrow \text{ Singular Matrix} \\ \det = 0$$

How to find the determinant using the calculator?

Once you've entered the matrix into the calculator...



$$\begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & -1 \\ 3 & 0 & 1 \end{bmatrix} \det = -8$$

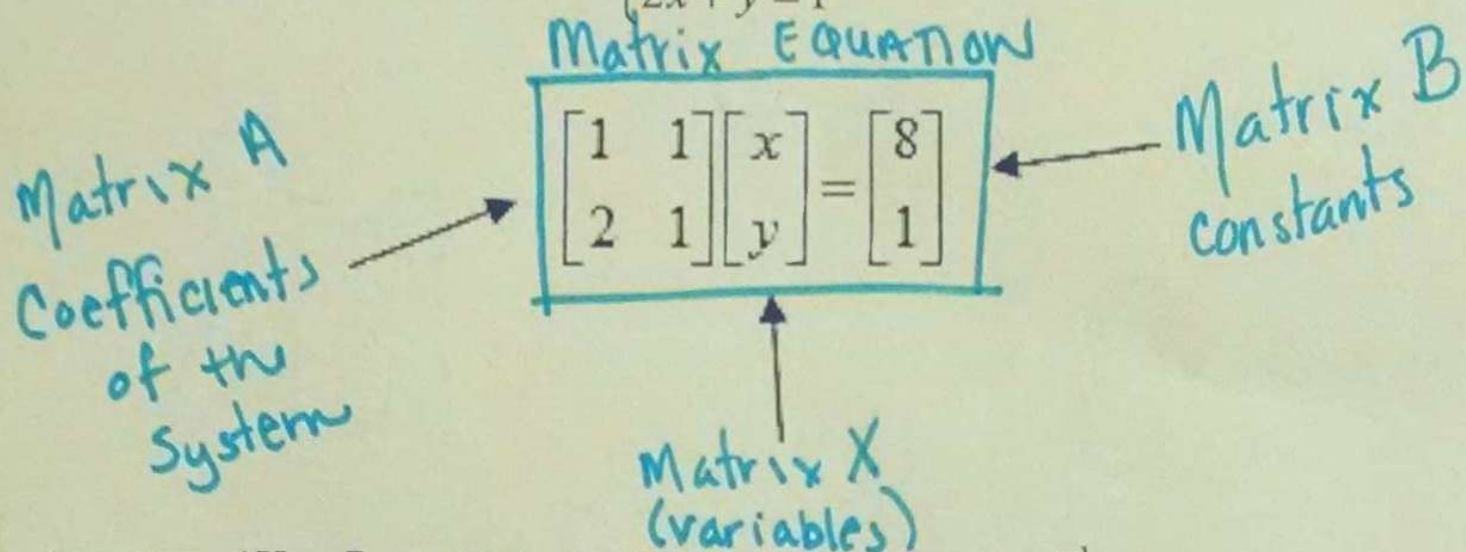
$$\begin{bmatrix} 8 & 2 \\ 4 & -1 \end{bmatrix} = -16$$

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} = -10$$

$$\begin{bmatrix} -6 & 3 \\ 9 & -5 \end{bmatrix} = 3$$

Using Inverse Matrices to Solve Systems

The matrix equation representing $\begin{cases} x + y = 8 \\ 2x + y = 1 \end{cases}$ is shown.



To solve $AX = B$, multiply both sides by the inverse A^{-1} .

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

Solution: (Inverse of A) • B

$$Ax + By = C$$

EQUATIONS MUST BE IN STANDARD FORM

1) $2x + 5y = 0$
 ~~$3x = 5x + 31$~~ $-5x + 3y = -31$

$$\begin{bmatrix} 2 & 5 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -31 \end{bmatrix}$$

A X B

$$A^{-1}B = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$$x = 5 \quad y = -2$$

2) $13x - y = 8$

~~$2x = 4$~~
 $-2x + 0y = -4$

$$\begin{bmatrix} 13 & -1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

$$x = \begin{bmatrix} 2 \\ 18 \end{bmatrix}$$

$$\text{a) } \begin{cases} x+y=8 \\ 2x+y=1 \end{cases}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$x = \begin{bmatrix} -7 \\ 15 \end{bmatrix}$$

$$\text{b) } \begin{cases} y = -x + 4 \\ 2x = 9 - 3y \end{cases}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\text{c) } \begin{cases} 3a + 6b - 2c = -6 \\ 2a + b + 4c = 19 \\ -5a - 2b + 8c = 62 \end{cases}$$

$$\begin{bmatrix} 3 & 6 & -2 \\ 2 & 1 & 4 \\ -5 & -2 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -6 \\ 19 \\ 62 \end{bmatrix}$$

$$x = \begin{bmatrix} -4 \\ 3 \\ 6 \end{bmatrix}$$

$$\text{d) } \begin{cases} x + 2y = 12 \\ 3y - 4z = 25 \\ x + 6y + z = 20 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -4 \\ 1 & 6 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 25 \\ 20 \end{bmatrix}$$

$$x = \begin{bmatrix} 6 \\ 3 \\ -4 \end{bmatrix}$$

GLUE HERE