Notes Solving Systems of Equations by Substitution (Replacement)

How would you solve the following system of equations?

$$\begin{cases} 3x + 2y = 11 \\ y = -2 \end{cases} ?$$

$3x + 2(-2) = 11$
$3x - 4 = 11$
$3x = 15$
$x = 5$
$(5, -2)$

To be used if one of the equations is, or can easily be, solved for a variable.

Step 1: Choose the equation to use as the substitute.	$\begin{cases} 3x + y = 10 \\ y = 2x \end{cases}$ $(2, 4)$
Step 2: Replace the variable with its equivalent in order to only have one variable in the new equation	$3x + 2x = 10$
Step 3: Solve by substitution	$5x = 10$ $x = 2$
Step 4: Substitute your answer back into one of the equations to find the other variable.	$y = 2x$ $y = 2(2)$ $y = 4$
Step 5: Now you should have an (x, y). Check your solution by plugging it in to original equations.	Check: $3(2) + 4 = 10$ $6 + 4 = 10$ $10 = 10$ ✓ $4 = 2(2)$ $4 = 4$ ✓

1) $\begin{cases} x + 2y = 8 \\ x = y - 1 \end{cases}$

$(y - 1) + 2y = 8$
$3y - 1 = 8$
$3y = 9$
$y = 3$
$x = 3 - 1$
$x = 2$
$(2, 3)$
check: $2 + 2(3) = 8$
$8 = 8$ ✓
$2 = 3 - 1$ ✓

2) $\begin{cases} x = 2y + 3 \\ x = y + 10 \end{cases}$

$(17, 7)$
$y + 10 = 2y + 3$
$y = 7$
$x = 7 + 10$
$x = 17$

3) $\begin{cases} 5x + 2y = 9 \\ y = x + 8 \end{cases}$

$5x + 2(x + 8) = 9$
$5x + 2x + 16 = 9$
$7x + 16 = 9$
$7x = -7$
$x = -1$
$y = -1 + 8$
$y = 7$
$(-1, 7)$

4) $\begin{cases} 3x + y = 5 \\ 5x - 4y = -3 \end{cases}$ $y = -3x + 5$

$5x - 4(-3x + 5) = -3$
$5x + 12x - 20 = -3$
$17x - 20 = -3$
$17x = 17$
$x = 1$
$3(1) + y = 5$
$y = 2$
$(1, 2)$

5) $\begin{cases} 2x + 3y = 7 \\ y = 6x - 11 \end{cases}$

$2x + 3(6x - 11) = 7$
$2x + 18x - 33 = 7$
$20x - 33 = 7$
$20x = 40$
$x = 2$
$y = 6(2) - 11$
$y = 12 - 11$
$y = 1$
$(2, 1)$

6) $\begin{cases} x = y + 1 \\ 3x + 3y = -3 \end{cases}$

$3(y + 1) + 3y = -3$
$3y + 3 + 3y = -3$
$6y + 3 = -3$
$y = -1$
$x = -1 + 1$
$x = 0$
$(0, -1)$

Notes: Solving Systems by ELIMINATION

Use when the information is in Standard Form (Ax + By = C)
There are three different situations you can come across.

Same Coefficient- Different sign 1) $\begin{cases} -4x-2y=-12 \\ 4x+8y=-24 \end{cases}$ Add the equations	$-4x-2y=-12$ $+\ 4x+8y=-24$ $6y=-36$ $y=-6$	$4x+8(-6)=-24$ $4x-48=-24$ $4x=24$ $x=6$ $(6,-6)$ check: $-4(6)-2(-6)=-12$ $-24+12=-12$ ✓ $4(6)+8(-6)=-24$ $24-48=-24$ ✓
Same Coefficient- Same Sign 2) $\begin{cases} 8x+2y=30 \\ 7x+2y=24 \end{cases}$ Multiply 1 Equation by -1, then add equations	$(8x+2y=30)-1$ $7x+2y=24$ $+\ -8x-2y=-30$ $-x=-6$ $x=6$	$8(6)+2y=30$ $48+2y=30$ $2y=-18$ $y=-9$ $(6,-9)$ check: $8(6)+2(-9)=30$ $48-18=30$ $30=30$ ✓ $7(6)+2(-9)=24$ $42-18=24$ $24=24$ ✓
Different Coefficients 3) $\begin{cases} 10x-7y=-18 \\ 5x+y=9 \end{cases}$ Multiply 1 or BOTH Equations to find LCM w/ opp signs then add equations	$10x-7y=-18$ $(5x+y=9)-2$ $-10x-2y=-18$ $+\ 10x-7y=-18$ $-9y=-36$ $y=4$	$5x+4=9$ $5x=5$ $x=1$ $(1,4)$ check: $10(1)-7(4)=-18$ $10-28=-18$ $-18=-18$ ✓ $5(1)+4=9$ $9=9$ ✓

Solve by Elimination:

$\begin{cases} 6x+4y=-10 \\ -6x+5y=1 \end{cases}$ $+$ $9y=-9$ $y=-1$ $6x+4(-1)=-10$ $6x-4=-10$ $6x=-6$ $x=-1$ $(-1,-1)$	$(-6x+3y=-12)-1$ $-6x+6y=6$ $6x-3y=12$ $3y=18$ $y=6$ $-6x+3(6)=-12$ $-6x+18=-12$ $-6x=-30$ $x=5$ $(5,6)$	$(3x-2y=2)5$ $(5x-5y=10)-3$ $15x-10y=10$ $+\ -15x+15y=-30$ $5y=-20$ $y=-4$ $3x-2(-4)=2$ $3x+8=2$ $3x=-6$ $x=-2$ $(-2,-4)$

More Practice… Yeah!!

1) $\begin{cases} x - y = 11 \\ 2x + y = 19 \end{cases}$ $(10, -1)$	2) $\begin{cases} 4x + 8y = 20 \\ -4x + 2y = -30 \end{cases}$ $(7, -1)$
3) $\begin{cases} 8x + y = -16 \\ -3x + y = -5 \end{cases}$ $(-1, -8)$	4) $\begin{cases} 7x + 2y = 24 \\ 8x + 2y = 30 \end{cases}$ $(6, -9)$
5) $\begin{cases} -4x + 9y = 9 \\ x - 3y = -6 \end{cases}$ $(9, 5)$	6) $\begin{cases} -7x - 8y = 9 \\ -4x + 9y = -22 \end{cases}$ $(1, -2)$
7) $\begin{cases} 5x + 4y = -30 \\ 3x - 9y = -18 \end{cases}$ $(-6, 0)$	8) $\begin{cases} 2x + 8y = 6 \\ -5x - 20y = -15 \end{cases}$ Infinitely many solutions