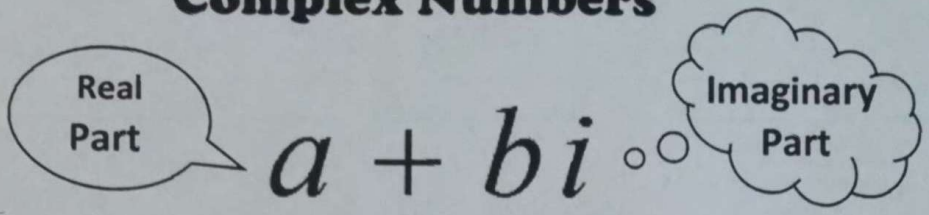
Complex Numbers



Imaginary Number: the square root of a negative number

$\sqrt{-1} = i$	$i^2 = -1$	$i^3 = -i$	$i^4 = 1$

Simplifying Rational and Complex Numbers (Pull out i's first, then simplify Radicals)

1. A) $\sqrt{25} = 5$

   B) $\sqrt{-25} = \sqrt{-1}\sqrt{25}$
   $i\sqrt{25} \Rightarrow 5i$

2. $\sqrt{-48}$
   $\sqrt{-1}\sqrt{48}$
   $i\sqrt{48}$
   $\sqrt{16}\sqrt{3}$
   $4i\sqrt{3}$

3. $-4\sqrt{-5}$
   $-4\sqrt{-1}\sqrt{5}$
   $-4i\sqrt{5}$

4. $5\sqrt{-98}$
   $5\sqrt{-1}\sqrt{98}$
   $5i\sqrt{98}$
   $\sqrt{49}\sqrt{2}$
   $5i \cdot 7\sqrt{2} \Rightarrow 35i\sqrt{2}$

5. $\sqrt{-24} \cdot \sqrt{-6}$
   $i\sqrt{24} \cdot i\sqrt{6}$
   $i \cdot i\sqrt{144}$
   $i^2\sqrt{144}$
   $12i^2 = 12(-1) \Rightarrow -12$

6. $\sqrt{-6} \cdot \sqrt{-6}$
   $i\sqrt{6}\ i\sqrt{6}$
   $i^2 \cdot \sqrt{36}$
   $6i^2 = 6(-1) \Rightarrow -6$

7. $-4\sqrt{-49}$
   $-4i\sqrt{49}$
   $-4i \cdot 7$
   $-28i$

8. $\frac{\sqrt{-8}}{\sqrt{2}}$ $\frac{i\sqrt{8}}{\sqrt{2}}$
   $i\sqrt{4} \Rightarrow 2i$

Simplifying Complex Numbers

Divide Exponent by 4 and look at *remainder*:

$.25 = i^1 = i$
$.5 = i^2 = -1$
$.75 = i^3 = -i$
No remainder : $i^4 = 1$

$i^{10/4}$	$i^{21/4}$	$i^{120/4}$	$i^{95/4}$
$i^{2.5}$	$i^{5.25}$	i^{30}	$i^{23.75}$
$i^2 = -1$	$i^1 = i$	$i^4 = 1$	$i^3 = -i$
i^{200}	i^{63}	i^{89}	i^{18}
1	$-i$	i	-1

Add and Subtract Complex Numbers (COMBINE LIKE TERMS)

$(5+2i)+(3-4i)$	$(8-11i)+(6-9i)$	$(3-10i)-(6+i)$	$(15+7i)-(7-3i)$
$8-2i$	$14-20i$	$3-10i-6-i$ $-3-11i$	$15+7i-7+3i$ $8+10i$

Multiply Complex Numbers (Remember to change any i^2 to (-1))

$6i(3-4i)$

$6i(5) - 6i(4i)$

$30i - 24i^2$

$30i - 24(-1)$

$30i + 24$

$(3-i)(4-i)$

$3\cdot4 \quad 3\cdot-i \quad -i\cdot4 \quad -i\cdot-i$

F O I L

$12 - 3i - 4i + i^2$

$12 - 7i + (-1)$

$11 - 7i$

$(2+3i)^2$

$(2+3i)(2+3i)$

$4 + 6i + 6i + 9i^2$

$4 + 12i + 9(-1)$

$4 + 12i - 9$

$-5 + 12i$

$(3+2i)(3-2i)$

$9 - 6i + 6i - 4i^2$

$9 - 4(-1)$

$9 + 4$

13

Conjugates

Rationalize Complex Numbers (NO IMAGINARY NUMBERS IN DENOMINATOR) or Radicals

Multiply by Denominator

$\frac{3}{i}\cdot\frac{i}{i} = \frac{3i}{i^2} = \frac{3i}{-1}$

$-3i$

$\frac{-9}{2i}\cdot\frac{2i}{2i} = \frac{-18i}{4i^2}$

$\frac{-18i}{4(-1)} = \frac{-18i}{-4}$

$\frac{9i}{2}$

$\frac{7}{i\sqrt{3}}\cdot\frac{i\sqrt{3}}{i\sqrt{3}} = \frac{7i\sqrt{3}}{i^2\sqrt{9}}$

$\frac{7i\sqrt{3}}{3i^2} \rightarrow \frac{7i\sqrt{3}}{-3}$

Like difference of 2 squares

Multiply by CONJUGATE

imaginary numbers have opposite signs

$\frac{5}{4+i}\frac{(4-i)}{(4-i)} = \frac{5(4-i)}{(4+i)(4-i)}$

$\frac{20-5i}{16-i^2} \rightarrow \frac{20-5i}{16+1}$

$\frac{20-5i}{17}$

$\frac{5i}{3-i}\frac{(3+i)}{(3+i)} = \frac{5i(3+i)}{(3-i)(3+i)}$

$\frac{15i+5i^2}{9-i^2} \rightarrow \frac{15i+5(-1)}{9+1}$

$\rightarrow \frac{15i-5}{10} \rightarrow \frac{3i}{2} - \frac{1}{2}$

$\frac{2-i}{5+3i}\frac{(5-3i)}{(5-3i)} \quad \frac{(2-i)(5-3i)}{(5+3i)(5-3i)}$

$\frac{10-6i-5i+3i^2}{25-9i^2} = \frac{10-11i+3(-1)}{25-9(-1)}$

$\rightarrow \frac{10-11i-3}{25+9} = \frac{7-11i}{34}$

Solving Quadratics with Complex Numbers (REARRANGE to the form $x^2 = c$)

When solving - don't forget ± signs!!

$\sqrt{x^2} = \sqrt{-81}$

$x = \sqrt{-81}$

$x = \sqrt{-1}\sqrt{81}$

$x = i\sqrt{81}$

$x = \pm 9i$

$x^2 + 48 = 0$

$\sqrt{x^2} = \sqrt{-48}$

$x = \sqrt{-48}$

$x = \sqrt{-1}\sqrt{48}$

$x = i\sqrt{16}\sqrt{3}$

$x = \pm 4i\sqrt{3}$

$3x^2 + 135 = 0$

$\frac{3x^2}{3} = \frac{-135}{3}$

$\sqrt{x^2} = \sqrt{-45}$

$x = \sqrt{-1}\sqrt{45}$

$x = \pm 3i\sqrt{5}$

$\sqrt{(x-2)^2} = \sqrt{-20}$

$x - 2 = \sqrt{-20}$

$x - 2 = \sqrt{-1}\sqrt{20}$

$x - 2 = \pm 2i\sqrt{5}$

$x = 2 \pm 2i\sqrt{5}$

$x^2 = -49$

$x = \pm 7i$

$x^2 + 72 = 0$

$x = \pm 6i\sqrt{2}$

$5x^2 + 160 = 0$

$x = \pm 4i\sqrt{2}$

$(x+5)^2 = -24$

$x = -5 \pm 2i\sqrt{6}$

GLUE HERE