Solving Quadratics by Completing the Square

Create a perfect square trinomial

$x^2 \frac{-16x}{2} + \frac{64}{(-8)^2}$	$x^2 \frac{-12x}{2} + \frac{36}{(-6)^2}$	$x^2 \frac{-2x}{2} + \frac{1}{(-1)^2}$	$x^2 \frac{+14x}{2} + \frac{49}{(7)^2}$
$(x-8)^2$	$(x-6)^2$	$(x-1)^2$	$(x+7)^2$

How to SOLVE by Completing the Square Use when in the form $(x^2 + bx = c)$

$x^2 - 10x - 50 = 0$
$+50 \quad +50$

$x^2 - 10x = 0 + 50$

$x^2 - 10x + 25 = 0 + 50 + 25$

$\sqrt{(x-5)^2} = \sqrt{75}$

$x - 5 = \pm 5\sqrt{3}$
$+5 \quad +5$

$x = 5 \pm 5\sqrt{3}$

Step 1: Is the function in the form $x^2 + bx = c$?
IF NOT, move all terms without variable to the other side

Step 2: Build a Perfect Square Trinomial

Step 3: Add the new "c" to both sides

Step 4: Factor and simplify

Step 5: Take Square Root of BOTH Sides and solve for x.

Don't Forget ± signs!

1) $x^2 - 14x = 0$

$x^2 - 14x + 49 = 0 + 49$

$\sqrt{(x-7)^2} = \sqrt{49}$

$x - 7 = \pm 7$

$x = 7 \pm 7$

$7 + 7 = 14$

$7 - 7 = 0$

$x = 0, 14$

2) $x^2 + 20x - 30 = -9$
$+30 \quad +30$

$x^2 + 20x = 21$

$x^2 + 20x + 100 = 21 + 100$

$\sqrt{(x+10)^2} = \sqrt{121}$

$x + 10 = \pm 11$

$x = -10 \pm 11$

$x = 1, -21$

3) $x^2 - 16x = 26$

$x^2 - 16x + 64 = 26 + 64$

$\sqrt{(x-8)^2} = \sqrt{90}$

$x - 8 = \pm 3\sqrt{10}$

$x = 8 \pm 3\sqrt{10}$

a) $x^2 + 18x = -1$

$x = -9 \pm 4\sqrt{5}$

b) $x^2 + 2x + 87 = -4$

$x = -1 \pm 3i\sqrt{10}$

c) $x^2 + 16x - 48 = 0$

$x = -8 \pm 4\sqrt{7}$

d) $x^2 - 12x = -9$

$x = 6 \pm 3\sqrt{3}$

$y = (x-h)^2 + k$

Writing equation of a Parabola in VERTEX Form by Completing the Square

$x^2 - 4x - 12 = y$

Which term is squared? x Move all terms WITHOUT x the other side

$x^2 - 4x = y + 12$

Complete the Square on x^2 side (Remember to ADD constant to BOTH sides)

$x^2 - 4x + 4 = y + 12 + 4$

FACTOR x^2 side (perfect square trinomial) and combine like terms on other side

$(x-2)^2 = y + 16$

Set equation equal to y

$(x-2)^2 - 16 = y$

Vertex $(2, -16)$

Rewrite the parabola in standard form by completing the square.

a) $y = x^2 - 10x - 75$

$y + 75 = x^2 - 10x$

$y + 75 + 25 = x^2 - 10x + 25$

$y + 100 = (x-5)^2$

$y = (x-5)^2 - 100$

Vertex: $(5, -100)$

b) $x^2 - 2x + 8 = y$

$x^2 - 2x = y - 8$

$x^2 - 2x + 1 = y - 8 + 1$

$(x-1)^2 = y - 7$

$(x-1)^2 + 7 = y$

Vertex: $(1, 7)$

c) $x^2 + 14x - 10 = y$

$x^2 + 14x = y + 10$

$x^2 + 14x + 49 = y + 10 + 49$

$(x+7)^2 = y + 59$

$(x+7)^2 - 59 = y$

Vertex: $(-7, -59)$

d) $f(x) = x^2 - 4x + 9$

$f(x) - 9 = x^2 - 4x$

$f(x) - 9 + 4 = x^2 - 4x + 4$

$f(x) - 5 = (x-2)^2$

$f(x) = (x-2)^2 + 5$

Vertex: $(2, 5)$

What happens when $a \neq 1$? move non "x" terms to other side, then Factor out "a"

a) $2x^2 + 14x - 10 = y$

$2x^2 + 14x = y + 10$ (2)(12.25)

$2(x^2 + 7x + 12.25) = y + 10 + 24.5$

$2(x+3.5)^2 = y + 34.5$

$2(x+3.5) - 34.5 = y$

b) $4x^2 - 16x + 2 = y$

$4x^2 - 16x = y - 2$ (4)(4)

$4(x^2 - 4x + 4) = y - 2 + 16$

$4(x-2)^2 = y + 14$

$y = 4(x-2) - 14$

c) $3x^2 + 12x + 3 = y$

$3x^2 + 12x = y - 3$

$3(x^2 + 4x + 4) = y - 3 + 4$

$3(x+2)^2 = y + 1$

$3(x+2)^2 - 1 = y$

$(-2, 1)$

d) $6x^2 - 24x - 5 = y$

$6x^2 - 24x = y + 5$

$6(x^2 - 4x + 4) = y + 5 + 24$

$6(x-2)^2 = y + 29$

$6(x-2) - 29 = y$

$(2, -9)$