POLYNOMIALS

A **POLYNOMIAL** is a number or **PRODUCT** of **NUMBERS** and **VARIABLES** with **whole** number exponents.

EXAMPLE	NON-EXAMPLES	DESCRIPTION
$3x^4$	3^{x}	Exponent cannot be a variable
$2z^{12}+9z^3$	$\lvert 2b^3-6b \rvert$	No Absolute Value
$\frac{1}{2}x^4$	$\frac{3}{5y^2}$	Variables cannot be in denominator
$0.15x^{101}$	$\frac{1}{2}\sqrt{x}$	variable cannot be radicant
$3x^4-x^5$	$x^{0.75}-x$	Exponents must be whole #

Terms are separated by + and - signs

Polynomials by TERM		
Specific name	Term	Example
monomial	1	3 , $3x$, $3x^5y^3$
Binomial	2	$3+x$, $3x^5+2y^9$
Trinomial	3	x^2+3x-2
~~quadnomial~~ polynomial	4+	x^3+3x^2+2y-5

Each monomial in a polynomial is a TERMS.

Polynomials by DEGREE		
Specific name	Degree	Example
Constant	0	$-9x^0$
Linear	1	x^1-9
Quadratic	2	x^2+x-9
Cubic	3	x^3+x^2+x-9
Quartic	4	$x^4+x^3+x^2+x-9$

Quintic 5

The DEGREE of a **MONOMIAL** is the Sum of the exponents. **Identify the degree of each monomial.**

A) $3x^4$ 4	B) 12 0
C) $3x^4y^1$ 5	D) $-5x^3y^4z^1$ 8

The DEGREE of a **POLYNOMIAL** is the degree of the Leading Term. **Identify the degree of each polynomial.**

A) $5x^4-6x^2+x-9$ 4	B) $-2x^7-3x^6+x^2-1$ 7
C) $5+7x-4x^2-x^3$ $-x^3-4x^2+7x+5$ 3	D) $6x^5+7x^4-2x^8+5x$ $-2x^8+6x+7x^4+5x$ 8

STANDARD FORM for a POLYNOMIAL

Largest degree ⟶ smallest

$$2x^6+7x^5-9x^4+3x^2-x+8$$

Leading coefficient 2 Degree 6 Terms 6

A) $2x+4x^2-1$ Quadratic Trinomial

Standard Form	LC	Degree	Terms
$4x^2+2x-1$	4	2	3

B) $7x^3-11x+x^5-2$

Standard Form	LC	Degree	Terms
$x^5+7x^3-11x-2$	1	5	4

Quintic Polynomial

C) $4x-2x^3+2$

Standard Form	LC	Degree	Terms
$-2x^3+4x+2$	-2	3	3

Cubic Trinomial

Find each Product: Show your work!!

A) $(x-2)(1+3x-x^2)$

$x+3x^2-x^3-2-6x+2x^2$

$-x^3+5x^2-5x-2$

B) $(x^2+3x-5)(x^2-x+1)$

C) $(3x-2y)(3x^2-xy-2y^2)$

D) $(2x+5)^2$

$(2x+5)(2x+5)$

$4x^2+10x+10x+25$

$4x^2+20x+25$

E) $(x-4)^3$

$(x-4)(x-4)(x-4)$

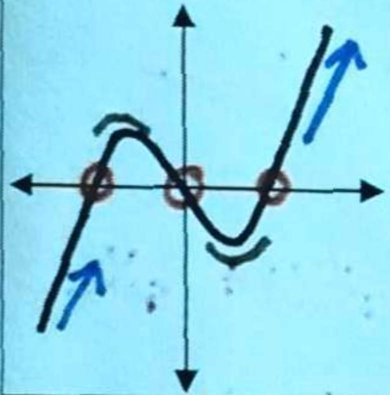
$(x^2-8x+16)(x-4)$

$x^3-4x^2-8x^2+32x+16x-64$

$x^3-12x^2+48x-64$

Graph each polynomial on a calculator. You may need to change the window. Describe the graph and identify the number of real zeros.

1. $f(x) = x^3 - x$



END BEHAVIOR

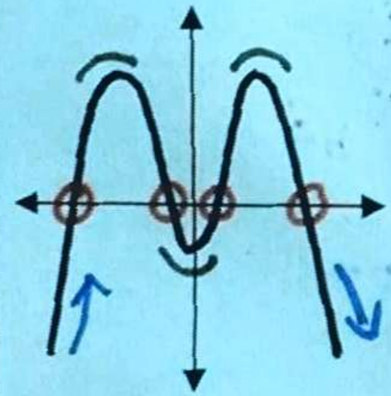
STARTS Increasing or decreasing and

ENDS Increasing or decreasing.

It crosses the x-axis 3 times, so there appears to be 3 real solutions.

There are 2 changes in direction.

2. $h(x) = -x^4 + 8x^2 - 1$



END BEHAVIOR

STARTS Increasing or decreasing and

ENDS Increasing or decreasing.

It crosses the x-axis 4 times, so there appears to be 4 real solutions.

There are 3 changes in direction.

Add Polynomials

1. $(3x^2 + 7 + x) + (14x^3 + 2 + x^2 - x)$

$3x^2 + 7 + x + 14x^3 + 2 + x^2 - x$

$4x^2 + 9 + 14x^3$

$14x^3 + 4x^2 + 9$

Subtract Polynomials

2. $(1 - x^2 + 2x) - (3x^2 + 2x - 6)$

$1 - x^2 + 2x - 3x^2 - 2x + 6$

$7 - 4x^2$

$-4x^2 + 7$

Review Exponent

Adding	$3x+5x$ $= 8x$	Add like terms Exponent don't change
Multiplying	$(2x^5)(-4x^7)$ $-8x^{12}$	multiply coefficients Add exponents
Powers	$(3x^3y^5)^4$ $3^4x^{12}y^{20} = 81x^{12}y^{20}$	power of coeff multiply exponents
Negative Exponents	$\frac{-2x^{-3}y^2}{1}$ $\frac{-2y^2}{x^3}$	neg exponents change position

A) $3x^2(x^3+4)$

$3x^5 + 12x^2$

B) $ab(a^3+3ab-b^3)$

$a^4b + 3a^2b^2 - ab^4$

$ab(a^3) + ab(3ab) + ab(-b^3)$

C) $2x^2y(6y^3+y^2-28y+30)$

$12x^2y^4 + 2x^2y^3 - 56x^2y^2 + 60x^2y$

To multiply any two polynomials, use the **DISTRIBUTIVE <u>Property</u>** and multiply each term in the second polynomial by each term in the first.

$(x+2)(x^2+4x-3)$

$$\begin{array}{r} x^3 + 4x^2 - 3x \\ + \quad + 2x^2 + 8x - 6 \\ \hline x^3 + 6x^2 + 5x - 6 \end{array}$$

OR the BOX Method

	x^2	$+4x$	-3
x	x^3	$+4x^2$	$-3x$
$+2$	$+2x^2$	$+8x$	-6

$x^3 + 6x^2 + 5x - 6$

Factoring the Sum and Difference of Cubes

Sum of Two Cubes	Difference of Two Cubes
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Same Opposite Always Positive

Factor Each Expression:

A) $y^3 - 27$

$(a-b)(a^2+ab+b^2)$

$\underset{a}{(y)^3} - \underset{b}{(3)^3}$

$(y-3)(y^2+3y+9)$

$y \cdot y \quad y \cdot 3 \quad 3^2$

B) $y^3 + 64$

$(y+4)(y^2-4y+16)$

C) $125x^3 + 1$

$(a+b)(a^2-ab+b^2)$

$\underset{a}{(5x)^3} + \underset{b}{(1)^3}$

$(5x+1)(25x^2-5x+1)$

$(5x)^2 \quad -5 \cdot x \quad (1)^2$

D) $8y^3 - 27$

$(2y)^3 - (3)^3$

$(2y-3)(4y^2+6y+9)$

C) $5x^4 + 40x$

$(a+b)(a^2-ab+b^2)$

$5x(x^3+8)$

$(x)^3 + (2)^3$

$(x+2)(x^2-2x+4)$

$5x(x+2)(x^2-2x+4)$

D) $16y^4 - 128y$

$16y(y^3-8)$

$(y)^3 - (2)^3$

$16y(y-2)(y^2+2y+4)$