Synthetic Division

use long division

$$\begin{array}{r} 156 \\ 4\overline{)624} \\ -4 \\ \hline 22 \\ -20 \\ \hline 24 \\ -24 \\ \hline 0 \end{array} \qquad \begin{array}{r} 121\tfrac{7}{8} \\ 8\overline{)975}\phantom{\tfrac{7}{8}} \\ -8\phantom{\tfrac{7}{8}} \\ \hline 17\phantom{\tfrac{7}{8}} \\ -16\phantom{\tfrac{7}{8}} \\ \hline 15\phantom{\tfrac{7}{8}} \\ -8\phantom{\tfrac{7}{8}} \\ \hline 7\phantom{\tfrac{7}{8}} \end{array}$$

Long Division

$$\frac{\text{dividend}}{\text{divisor}}$$

Long Division of Polynomials

Remember:

- Works every time
- WRITE Dividend IN STANDARD FORM (polynomial under division symbol)
- USE PLACE HOLDERS FOR MISSING TERMS (all degrees must be represented)
- DISTRIBUTE THE NEGATIVE

Ex: $\dfrac{x^3 - 2x^2 - 9}{x - 3}$ missing "x"

$$\begin{array}{r|l} & x^2 + x + 3 \\ x - 3 & x^3 - 2x^2 + 0x - 9 \\ & -(x^3 - 3x^2) \\ \hline & x^2 + 0x \\ & -(x^2 - 3x) \\ \hline & 3x - 9 \\ & -(3x - 9) \\ \hline & 0 \end{array}$$

$\frac{x^3}{x} = x^2$, $x^2(x-3)$

$\frac{x^2}{x} = x$, $x(x-3)$

$\frac{3x}{x} = 3$, $3(x-3)$

$\boxed{x^2 + x + 3}$

Ex: $(3x^4 - x^3 + 5x - 1) \div (x + 2)$ missing x^2

$$\begin{array}{r|l} & 3x^3 - 7x^2 + 14x - 23 + \frac{45}{x+2} \\ x + 2 & 3x^4 - x^3 + 0x^2 + 5x - 1 \\ & -(3x^4 + 6x^3) \\ \hline & -7x^3 + 0x^2 \\ & -(-7x^3 - 14x^2) \\ \hline & 14x^2 + 5x \\ & -(14x^2 + 28x) \\ \hline & -23x - 1 \\ & -(-23x - 46) \\ \hline & 45 \end{array}$$

$\frac{3x^4}{x} = 3x^3$

$\frac{-7x^3}{x} = -7x^2$

$\frac{14x^2}{x} = 14x$

$\frac{-23x}{x} = -23$

Synthetic Division of Polynomials

- Only works when divisor is LINEAR

Steps:

1) **Find the "zero" of divisor (set divisor equal to zero and solve)**
2) **List coefficients of dividend in standard form (with placeholders)**
3) **Bring Leading Coefficient (first #) straight down**
4) **Multiply "zero" with the number you just brought down and write product under next coefficient**
5) **Add the two numbers together and write sum in bottom row.**
6) **Repeat steps 4 and 5 until all columns are completed.**

Ex: $\dfrac{3x^3 - 8x^2 + 4}{x - 2}$ $0x$

$x - 2 = 0$
$x = 2$

$$\begin{array}{r|rrr|r} 2 & 3 & -8 & 0 & 4 \\ & \downarrow & 6 & -4 & -8 \\ \hline & 3_{x^2} & -2_{x} & -4 & -4 \end{array}$$

remainder

constant

$\boxed{3x^2 - 2x - 4 + \frac{-4}{x-2}}$

Ex: $(3x^4 - x^3 + 5x - 1) \div (x + 2)$ $0x^2$

$x + 2 = 0$
$x = -2$

$$\begin{array}{r|rrrr|r} -2 & 3 & -1 & 0 & 5 & -1 \\ & \downarrow & -6 & 14 & -28 & 46 \\ \hline & 3_{x^3} & -7_{x^2} & 14_{x} & -23 & 45 \end{array}$$

$3x^3 - 7x^2 + 14x - 23 + \frac{45}{x+2}$

Use Synthetic Substitution to evaluate the polynomial.

$P(x) = 4x^2 - 9x + 2$, when $P(3)$

$P(3) = 4(3)^2 - 9(3) + 2$

$36 - 27 + 2$

$P(3) = 11$

Use Synthetic Substitution

$$\begin{array}{r|rr|r} 3 & 4 & -9 & 2 \\ & \downarrow & 12 & 9 \\ \hline & 4 & 3 & 11 \end{array}$$

$P(x) = -x^3 - 3x^2 + 10x - 4$, when $P(-2)$

$$\begin{array}{r|rrr|r} -2 & -1 & -3 & 10 & -4 \\ & \downarrow & 2 & 2 & -24 \\ \hline & -1 & -1 & 12 & -28 \end{array}$$

$P(-2) = -28$

Use Synthetic Division to determine if the binomial is a factor of the polynomial.

$P(x) = 2x^4 + 2x^3 - x^2 - 5x - 4$, $(x+1)$

$x = -1$

$$\begin{array}{r|rrrr|r} -1 & 2 & 2 & -1 & -5 & -4 \\ & \downarrow & -2 & 0 & 1 & 4 \\ \hline & 2 & 0 & -1 & -4 & 0 \end{array}$$

No Remainder

↳ $x+1$ is a factor

$P(x) = 5x^3 + x^2 - 7$, $(x-2)$

$x = 2$

$$\begin{array}{r|rrr|r} 2 & 5 & 1 & 0 & -7 \\ & \downarrow & 10 & 22 & 44 \\ \hline & 5 & 11 & 22 & 37 \end{array}$$

Remainder $\neq 0$

↳ $x-2$ is not a factor

No Remainder (0) $\Longrightarrow$ Factor

Remainder $\neq 0 \Longrightarrow$ Not a Factor