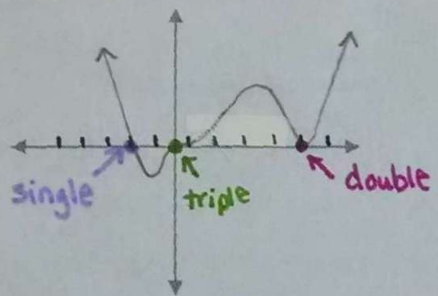
1) $x^3(x-5)^2(x+2)$

Set factor = 0 & solve

Factor	Root	Multiplicity
x^3	0	3
$x-5$	5	2
$x+2$	-2	1

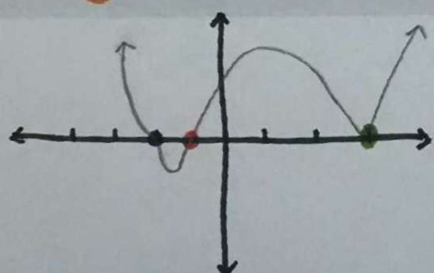


2) $(2x+1)(x+1)^3(x-3)^2$

$2x+1=0$ $\quad x+1=0$ $\quad x-3=0$

$x=-\frac{1}{2}$ $\quad x=-1$ $\quad x=3$

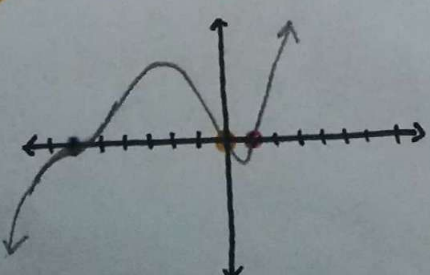
S $\quad$ T $\quad$ D



3) $x(x+6)^3(x-1)$

$x=0$ $\quad x=-6$ $\quad x=1$

S $\quad$ T $\quad$ S



1) $3x^5+18x^4+27x^3=0$

$3x^3(x^2+6x+9)=0$

$(x^2+3x)(+3x+9)$

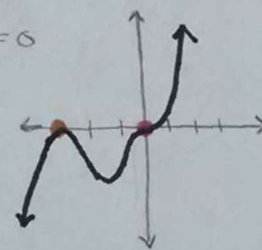
$x(x+3)+3(x+3)$

$3x^3(x+3)(x+3)=0$

$3x^3(x+3)^2=0$

$3x^3=0$ $\quad x+3=0$

$\underset{T}{x=0}$ $\quad \underset{D}{x=-3}$



2) $x^4-37x^2=-36$

$x^4-37x^2+36=0$

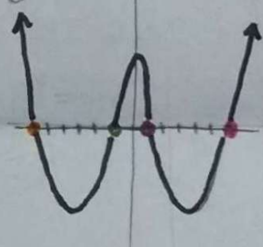
$(x^4-x^2)(-36x^2+36)=0$

$x^2(x^2-1)-36(x^2-1)=0$

$(x^2-36)(x^2-1)=0$

$(x+6)(x-6)(x+1)(x-1)=0$

$x=-6$ $\quad x=6$ $\quad x=-1$ $\quad x=1$



3) $2x^6-10x^5=12x^4$

$2x^6-10x^5-12x^4=0$

$2x^4(x^2-5x-6)=0$

$2x^4(x-6)(x+1)=0$

$x=0$ $\quad x=6$ $\quad x=-1$

1) $9x^3-23x^2-62x+40=0$ $\quad$ 3 roots

Roots from graph → -2, 4

$$\begin{array}{r|rrr|r} 4 & 9 & -23 & -62 & 40 \\ & \downarrow & 36 & 52 & -40 \\ \hline & 9x^2 & 13x & -10 & \varnothing \end{array}$$

$$\begin{array}{r|rr|r} -2 & 9 & 13 & -10 \\ & \downarrow & -18 & 10 \\ \hline & 9x & -5 & \varnothing \end{array}$$

$9x-5=0$

$+5 \quad +5$

$9x=5$

$x=5/9$

$\boxed{x=-2,\ 5/9,\ 4}$

3) $x^4+7x^3+63x+36=55x^2$

$x^4+7x^3-55x^2+63x+36=0$

Roots from Graph → -12, 3

$$\begin{array}{r|rrrr|r} 3 & 1 & 7 & -55 & 63 & 36 \\ & \downarrow & 3 & 30 & -75 & -36 \\ \hline & 1x^3 & 10x^2 & -25x & -12 & \varnothing \end{array}$$

$$\begin{array}{r|rrr|r} -12 & 1 & 10 & -25 & -12 \\ & & -12 & 24 & 12 \\ \hline & 1x^2 & -2x & -1 & \varnothing \end{array}$$

$x^2-2x-1=0$

$x^2-2x+\underline{1}=1+\underline{1}$

$\sqrt{(x-1)^2}=\sqrt{2}$

$x-1=\pm\sqrt{2}$

$x=1\pm\sqrt{2}$

$\boxed{x=-12,\ 3,\ 1\pm\sqrt{2}}$

Finding Roots of Polynomials in factored form & Graphing

(without a calculator)

1) Identify each factor
2) Find the root by setting = 0
3) Determine multiplicity from exponent(single, double, triple)
4) Graph

Finding the Roots of Polynomials by FACTORING

1) Set function equal to 0
2) Factor
3) Set each Factor equal to 0 and solve.
4) Sketch graph using roots and multiplicity.

Finding EXACT Roots of Polynomials (not factorable) using Calculator and Synthetic Division

1) Use Calculator to find 2 real roots that are integers (if possible)
2) Use Synthetic Division with 1 root to find factor.
3) Repeat Step 2 with other factors
4) Solve remaining function using factoring, square root or quadratic formula , or equation

Finding Roots of Polynomial Functions

Multiplicity single, double, and triples

In the example $3x^5 + 18x^4 + 27x^3 = 0$ has two *multiple* roots.

The **<u>MULTIPLICITY</u>** of a root r is the number of times that $x - r$ is a **<u>FACTOR</u>** of $P(x)$.

Multiplicity of 1 (SINGLE):
Graph ***CROSSES*** the x - axis.

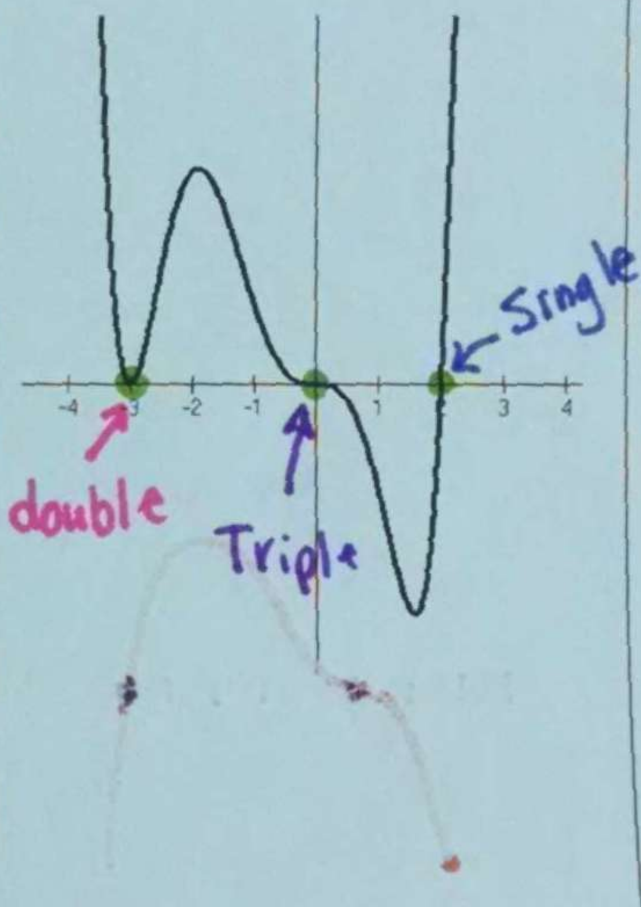
$(x-2) = (x-2)^1$

Multiplicity of 2 (DOUBLE):
Graph ***TOUCHES (Bounces)*** the x - axis but does not cross it.

$(x+3)^2$

Multiplicity of 3 (TRIPLE):
Graph ***"BENDS"*** as it crosses the x - axis.

x^3



GLUE HERE

Set Factor $=0$ and solve

1. Identify the roots of each function, the multiplicity.

a. $f(x)=x^2(2x+1)^1(x-3)^1$

Factor	Root	Multiplicity
$x-3$	3	1
$2x+1$	$-1/2$	1
x	0	2

b. $f(x)=(x+5)^3(x-6)^2$

Factor	Root	Multiplicity
$x+5$	-5	3
$x-6$	6	2

c. $f(x)=x(2x-5)^3(x+1)^2$

Factor	Root	Multiplicity
x	0	1
$2x-5$	$5/2$	3
$x+1$	-1	2

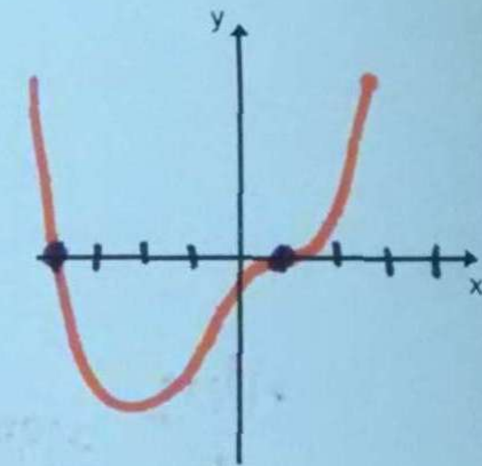
d. $f(x)=(x-2)(2x+1)^2(3x+1)^2$

Factor	Root	Multiplicity

2. Use your graphing calculator to make a sketch of the function and to find It roots and multiplicity. Then rewrite the function in factored form.

A) $f(x)=x^4+x^3-9x^2+11x-4$

Factor	Root	Multiplicity
$x+4$	-4	1
$x-1$	1	3

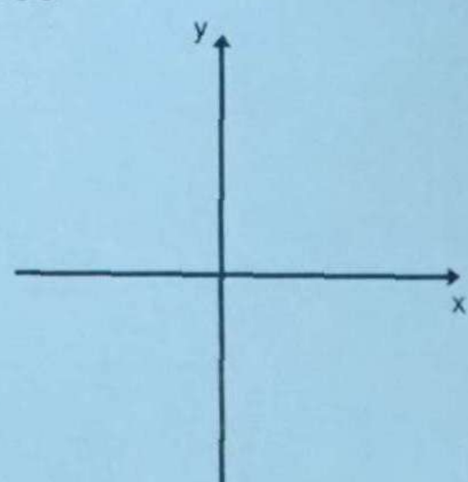


$f(x)=$ $(x+4)(x-1)^3$

$(x+4)(x-1)(x-1)(x-1)$

B) $f(x)=x^5+16x^4+x^3-470x^2+1276x-968$

Factor	Root	Multiplicity



$f(x)=$ ____________________

Write the **simplest function** (in factored form) with the following zeros.

A) -2, 2 and 4

$(x+2)$ $(x-2)$ $(x-4)$

$(x+2)(x-2)(x-4)$

B) $-5i$, 1 *and* 2

$x^2 = (\pm 5i)^2$

$x^2 = -25$

(x^2+25) $(x-1)$ $(x-2)$

$(x^2+25)(x-1)(x-2)$

GLUE HERE

Rational Root Theorem

If the polynomial $P(x)$ has integer coefficients, then every rational root of the polynomial equation $P(x)=0$ can be written in the form $\frac{p}{q}$, where $\boxed{p}$ is a factor of the constant term and $\boxed{q}$ is a factor of the leading coefficient of $P(x)$, written in standard form.

Use the Rational Root Theorem to identify ALL POSSIBLE rational roots.

$P(x) = \boxed{4}x^3 + x^2 - 8x + \boxed{12}$

$\frac{P}{Q}$ $\frac{12:}{4:}$ $\frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1, \pm2, \pm4}$

$\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm2, \pm3, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm4, \pm6, \pm12$

Irrational Root Theorem

** Irrational Roots come in pairs (the root and its conjugate)

Factor	Conjugate
$\sqrt{2}$	$-\sqrt{2}$
$(1-\sqrt{7})$	$1+\sqrt{7}$
$(3+2\sqrt{15})$	$3-2\sqrt{15}$

Use the irrational root theorem to find the smallest possible degree of the polynomials with the given roots.

A) -4 *and* $\sqrt{2}$, $-\sqrt{2}$	3
B) $2, -3$ *and* $1+\sqrt{2}$, $1-\sqrt{2}$	4
C) $2-\sqrt{5}, \sqrt{3},$ *and* $-4+\sqrt{3}$ $2+\sqrt{5}$ $-\sqrt{3}$ $-4-\sqrt{3}$	6

Complex Root Theorem

** Complex Roots come in pairs(the root and its conjugate)

Factor	Conjugate
$2i$	$-2i$
$(1-2i)$	$1+2i$
$(3+2i\sqrt{5})$	$3-2i\sqrt{5}$

Use the complex root theorem to find the smallest possible degree of the polynomials with the given roots.

A) $5i,\ 1,\ 2$ $-5i$	4
B) $\frac{1}{2},\ i,\ 1+\sqrt{3}$ $-i$ $1-\sqrt{3}$	5
C) $2i,\ 5+\sqrt{3}, 6$ $-2i$ $5-\sqrt{3}$	5