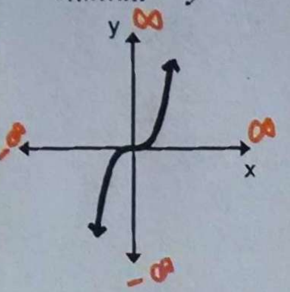
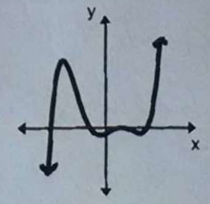
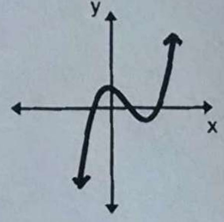
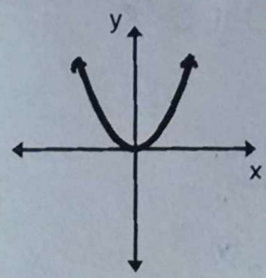
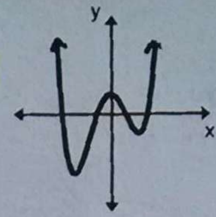
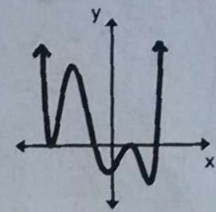
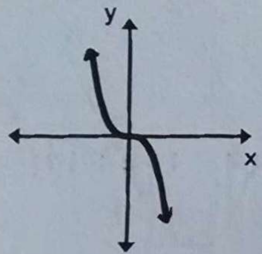
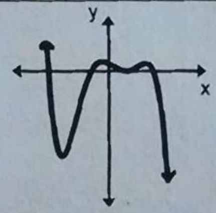
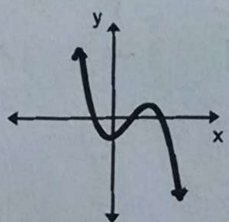
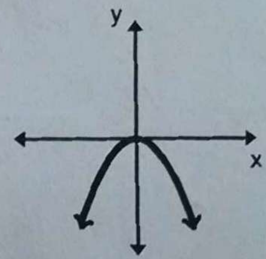
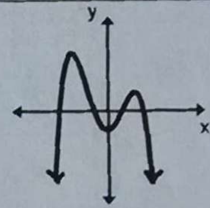
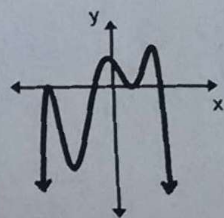
	ODD Degree (Opposite Direction)	**EVEN** Degree (Same Direction)
Positive Coefficient	Think... $y = x^3$ **+**    End behavior: $x \to -\infty$ $y \to$ $-\infty$; $x \to \infty$ $y \to$ ∞	Think... $y = x^2$ **+**    End behavior: $x \to -\infty$ $y \to$ ∞; $x \to \infty$ $y \to$ ∞
Negative Coefficient	Think... $y = -x^3$ **−**    End behavior: $x \to -\infty$ $y \to$ ∞; $x \to \infty$ $y \to$ $-\infty$	Think... $y = -x^2$ **−**    End behavior: $x \to -\infty$ $y \to$ $-\infty$; $x \to \infty$ $y \to$ $-\infty$

Multiplicity & End Behavior

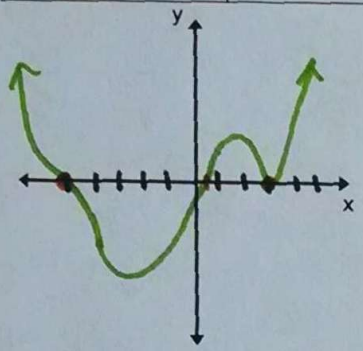
GLUE HERE ☺

WRITE THE EQUATION FOR EACH POLYNOMIAL IN FACTORED FORM.

1. Given that the following **Leading Coefficient is positive**, give the requested information, sketch a graph (no calculator) and write the function in factored form.

Factors	Roots	Multiplicity
$x-3$	3	2
$x+5$	-5	3
$2x-1$	1/2	1

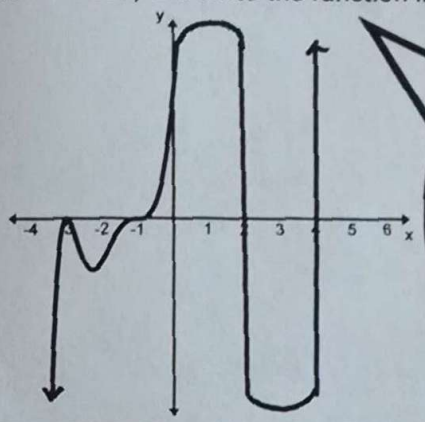
6 - even ↑↑



Degree: 6 End Behavior: $x \to -\infty$ $y \to \infty$; $x \to \infty$ $y \to \infty$

$P(x) = (x-3)^2(x+5)^3(2x-1)$

2. Given the following function, give the requested information, and write the function in factored form.



You need to ask yourself...?
degree: odd/even
coeff: pos/neg
multiplicity
· Roots
· Factors

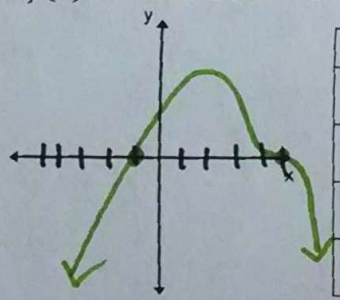
Factors	Roots	Multiplicity
$x+3$	-3	2
$x+1$	-1	3
$x-2$	2	1
$x-4$	4	1

Degree: 7 End Behavior: $x \to -\infty$ $y \to -\infty$; $x \to \infty$ $y \to \infty$

$P(x) = (x+3)^2(x+1)^3(x-2)(x-4)$

Find all solutions:

3. $f(x) = -x^4 + 14x^3 - 60x^2 + 50x + 125$

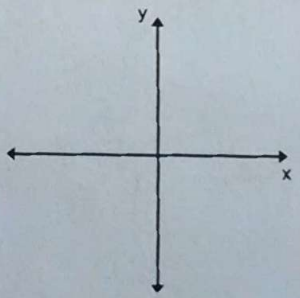


Factors	Roots	Mult.
$x+1$	-1	1
$x-5$	5	3

Degree: 4 End Behavior: $x \to -\infty$ $y \to -\infty$; $x \to \infty$ $y \to -\infty$

$P(x) = (x+1)(x-5)^3$

4. $f(x) = x^3 - 10x^2 + 25x - 6$



Factors	Roots	Mult.
$x-6$	6	1
		1
		1

6 | 1 -10 25 | -6
↓ 6 -24 | 4
1 x^2 - 4x 1 | ∅

$x^2 - 4x + 1 = 0$

$x^2 - 4x + 4 = -1 + 4$

$\sqrt{(x-2)^2} = \sqrt{3}$

$x - 2 = \pm\sqrt{3}$

$x = 2 \pm \sqrt{3}$

Degree: 3 End Behavior: $x \to -\infty$ $y \to -\infty$ ↓; $x \to \infty$ $y \to \infty$ ↑

Roots: $6, 2 \pm \sqrt{3}$

Fundamental Theorem of Algebra

Every polynomial function of degree $n \geq 1$ has at least one zero, where a zero may be a complex number

Corollary: Every polynomial function $n \geq 1$ has exactly n zeros, including multiplicities.

Glue Here

Solve each equation by finding all the roots

By Factoring

$x^4 - 625 = 0$

$(x^2 - 25)(x^2 + 25) = 0$

$(x-5)(x+5)(x^2+25) = 0$

$x-5=0 \quad x+5=0 \quad x^2+25=0$

$x=5 \quad x=-5 \quad \sqrt{x^2} = \sqrt{-25}$

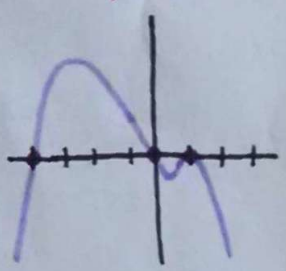
$x = \pm 5i$

$x = \pm 5, \pm 5i$

By graphing

$x^4 - 2x^3 - 7x^2 - 4x = 0$

$x(x+1)^2(x-4)$



$x = -1, 0, 4$

$x = -1, 0, 4$

By Factoring

$x^3 + 4x = 6x^2 + 24$

$(x^3 - 6x^2) + (4x - 24) = 0$

$x^2(x-6) + 4(x-6)$

$(x^2+4)(x-6) = 0$

$x^2+4=0 \quad x-6=0$

$\sqrt{x^2} = \sqrt{-4} \quad x=6$

$x = \pm 2i$

$x = 6, \pm 2i$

Exact Value using Synthetic Division

$x^4 + 3x^3 + 12x = 6x^2 + 40$

$x^4 + 3x^3 - 6x^2 + 12x - 40 = 0$

roots: $-5, 2$

2	1	3	-6	12	-40
		2	10	8	40
	1	5	4	20	$\emptyset$

-5	1	5	4	20
		-5	0	-20
	1	0	4	$\emptyset$

$x^2+4=0$

$x^2 = -4$

$x = \pm 2i$

$x = -5, 2, \pm 2i$

More Fun Practice for you ☺ No decimals allowed!!

$x^4 + 13x^2 = 48$

$x^4+13x^2-48=0$

$(x^4+16x^2-3x^2-48)=0$

$x^2(x^2+16)-3(x^2+16)=0$

$(x^2-3)(x^2+16)=0$

$x^2-3=0$ $\quad x^2+16=0$

$x=\pm\sqrt{3}$ $\quad \sqrt{x^2}=\sqrt{-16}$

$x=\pm 4i$

$x=\pm\sqrt{3}, \pm 4i$

$5x^4 + 11x^3 + 23x^2 + 77x - 84 = 0$

root $x=-3$ (from graph

-3	5	11	23	77	-84
	↓	-15	12	-105	84
	5	-4	35	-28	∅

$(5x^3-4x^2+35x-28)=0$

$x^2(5x-4)+7(5x-4)$

$x^2+7=0$ $\quad 5x-4=0$

$x=\pm i\sqrt{7}, \frac{4}{5}, -3$

$(x-3)(2x+1)^2=0$

$x=3 \quad x=-1/2$

$x^4 = 42 - x^2$

$x^4+x^2-42=0$

$(x^4+7x^2-6x^2-42)$

$x^2(x^2+7)-6(x^2+7)=0$

$(x^2-6)(x^2+7)=0$

$x^2-6=0$ $\quad x^2+7=0$

$x^2=6$ $\quad x^2=-7$

$x=\pm\sqrt{6}$ $\quad x=\pm i\sqrt{7}$

$x^3 + 4x^2 + 2x = 28$

$x^3+4x^2+2x-28$

root: 2

2	1	4	2	-28
		2	12	28
	1	6	14	∅

$x^2+6x+14=0$

$x^2+6x+\underline{9}=-14+\underline{9}$

$\sqrt{(x+3)^2}=\sqrt{-5}$

$x+3=\pm i\sqrt{5}$

$x=-3\pm i\sqrt{5}, 2$

$x^3 + 5x^2 - 9x - 45 = 0$

$x^2(x+5)-9(x+5)=0$

$(x^2-9)(x+5)$

$(x+3)(x-3)(x+5)=0$

$x=3, -3, -5$