CURVE FITTING WITH POLYNOMIAL MODELS & REGRESSION

The table shows the closing value of a stock index on the first day of trading for various years.

Year	1994	1995	1996	1997	2000	2001	2002	2003
Price ($)	774	751	1053	1293	4186	2474	1347	2011

To create a mathematical model for the data, you will need to determine what **type of function** is most appropriate. You learned that a set of data that has **constant second differences** can be modeled by a Quadratic function. Finite differences can be used to identify the **degree** of any polynomial data.

Finite Differences of Polynomials		
Function Type	**Degree**	**Constant Finite Differences**
Linear	1	First
Quadratic	2	Second
Cubic	3	Third
Quartic	4	Fourth
Quintic	5	Fifth

Example 1: Use finite differences to determine the degree of the polynomial that best describes the data.

a.

x	y
-2	-10
-1	-4
0	-1.4
1	0
2	2.4
3	8

1st: +6, +2.6, +1.4, +2.4, +5.6
2nd: -3.4, -1.2, +1
3rd: +2.2, +2.2, +2.2

Not Linear

NOT QUADRATIC

Cubic b/c 3rd diff is constant

b.

x	y
-6	-30
-4	15
-2	30
0	34
2	41
4	60

1st: +45, +15, +4, +7, +19
2nd: -30, -11, +3, +12
3rd: +19, +14, +9
4th: -5, -5

NOT LINEAR

NOT QUADRATIC

NOT CUBIC

Quartic b/c 4th diff is constant

c.

x	y
-2	-24
-1	-27
0	-28
1	-21
2	0
3	41

1st: -3, -1, +7, +21, +41
2nd: +2, +8, +14, +20
3rd: +6, +6, +6

NOT LINEAR

NOT QUADRATIC

Cubic b/c 3rd diff is constant

Once you have determined the degree of the polynomial that best describes the data, you can use your calculator to create the function.

Example 2: Word Problem.

The table shows the population of a city from 1950 to 2000. Write a polynomial function for the data.

	0	10	20	30	40	50
Year	1950	1960	1970	1980	1990	2000
Population (thousands)	2853	4011	5065	6720	9704	14759

Step 1: Find the finite differences of the y-values.

1ST: 1158, 1054, 1655, 2984, 5055

2nd: -104, 601, 1329, 2071

3rd: 705, 728, 742 → close enough

Step 2: Determine the degree of the polynomial.

3rd difference is relatively close

↳ Cubic model

Step 3: Use the regression feature on your calculator to find the function model for the data.

$y \approx .12x^3 - 711.57x^2 + 1395925.03x - 912875770.9$

To determine the best model using your calculator, use the regression feature of your graphing calculator. Make sure your **Diagnostics are turned on.** 2^{Nd} → 0 → scroll down to "DiagnosticsOn"→ Enter Enter

Remember → the closer the R^2 is to 1, the better the function fits the data.

The table shows the value of a stock index on the first day of trading in various years since 1994. Use a polynomial model to estimate the value on the first day of trading in 2002.

x = # of years after 1994

	0	1	2	3	6	7	9	10
Year	1994	1995	1996	1997	2000	2001	2003	2004
Price ($)	774	751	1053	1293	4186	2474	1347	2011

Linear: $R^2 \approx .26$

Quadratic: $R^2 \approx .55$

Cubic $R^2 \approx .63$

Quartic: $R^2 \approx .84$ → closest to 1

Quartic

$y \approx 9.27x^4 - 191.56x^3 + 1161.22x^2 - 1702.58x - 1702.58$

2002 → $2031

Glue Here