Notes: Using Transformations to Graph Quadratic Functions in Vertex Form

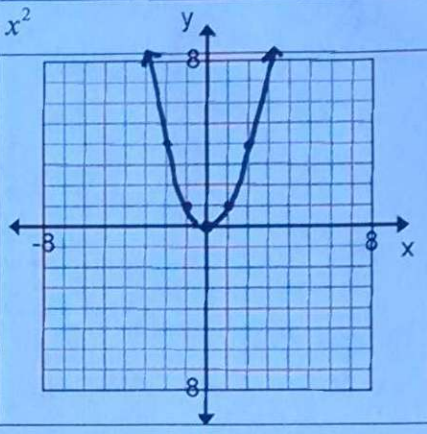
The Quadratic Parent Function $f(x) = x^2$

Domain: (x-values)
$\mathbb{R}$

Range: (y-values)
$y \geq 0$

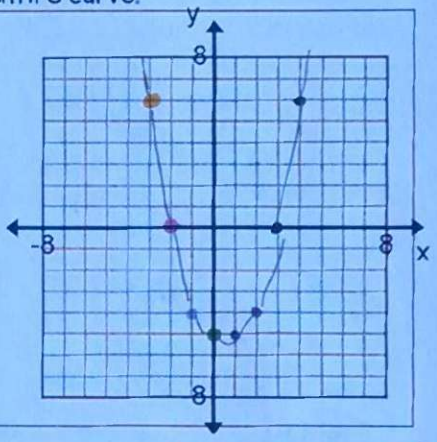
Vertex:
$(0,0)$

x	$y = x^2$
−4	16
−2	4
0	0
2	4
4	16



Graph $f(x) = x^2 - x - 6$ by using a table. Plot enough points to see the entire curve.

x	$f(x) = x^2 - x - 6$	$(x, f(x))$
-3	$(-3)^2 - -3 - 6$	(-3, 6)
-2	$(-2)^2 - -2 - 6$	(-2, 0)
-1	$(-1)^2 - -1 - 6$	(-1, -4)
0	$(0)^2 - 0 - 6$	(0, -6)
1	$(1)^2 - 1 - 6$	(1, -6)
2	$(2)^2 - 2 - 6$	(2, -4)



Identify how a quadratic function is transformed by changing a, h, and k

Changes width
$a > 1$ → vertical stretch (narrower)
$0 < a < 1$ → vertical compression (wider)

Vertex : (h, k)

Horizontal Translation
$(x+h)$ → h units LEFT
$(x-h)$ → h units RIGHT

$$f(x) = a(x-h)^2 + k$$

$a > 0$ → opens up
$a < 0$ → opens down (reflect across x-axis)

Vertical Translation
$x^2 + k$ → k units UP
$x^2 - k$ → k units DOWN

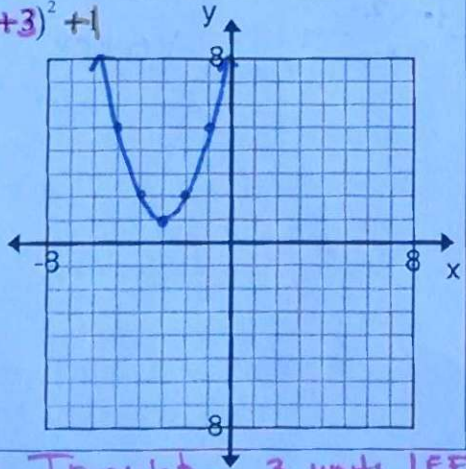
Without using a calculator, graph the transformation (use at least 3 points)

a. $g(x) = (x+3)^2 + 1$

Vertex
$(-3, 1)$

Domain:
$\mathbb{R}$

Range:
$y \geq 1$



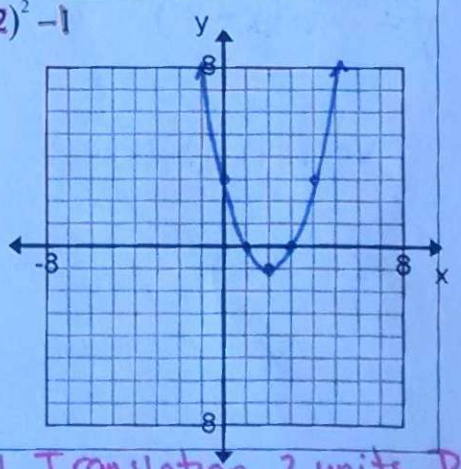
Horizontal Translation 3 units LEFT
Vertical Translation 1 unit UP

b. $g(x) = (x-2)^2 - 1$

Vertex
$(2, -1)$

Domain:
$\mathbb{R}$

Range:
$y \geq -1$



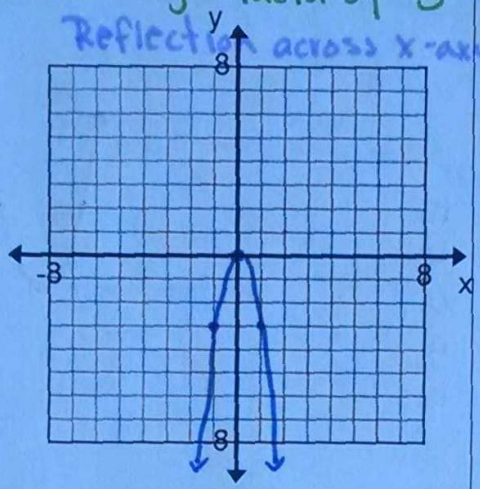
Horizontal Translation 2 units RIGHT
Vertical Translation 1 unit DOWN

c. $g(x) = -3x^2$

Vertical Stretch by a factor of 3

Reflection across x-axis

Vertex (0,0)



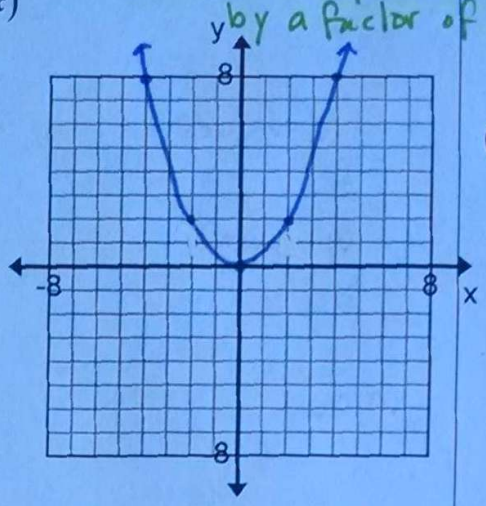
Domain: $\mathbb{R}$

Range: $y \leq 0$

d. $g(x) = \frac{1}{2}(x)^2$

(0,0)

Vertical Compression by a factor of 1/2



Domain: $\mathbb{R}$

Range: $y \geq 0$

This lowest or highest point is the vertex of a parabola. (where it changes direction)

If a parabola opens upward, it has a ∪ minimum point. (max or min?)

If a parabola opens downward, it has a ∩ maximum point. (max or min?)

The parent function $f(x) = x^2$ has its vertex at the (0,0) origin.

Because the vertex is translated h horizontal units and k vertical units from the origin, the vertex of the parabola is at (h, k).

Use the description to write the quadratic function in vertex form. Check w/ calculator. ☺

a. The parent function $f(x) = x^2$ is reflected across the x-axis, vertically stretched by a factor of 6, and translated 3 units right to create g.

$y = a(x-h)^2 + k$

$y = -6(x-3)^2$

Vertex: (3,0)

b. The parent function $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{3}$ and translated 2 units right and 4 units down to create g.

$y = \frac{1}{3}(x-2)^2 - 4$

c. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 5 units left and 1 unit up to create g.

$y = -(x+5)^2 + 1$

Vertex: (-5,1)