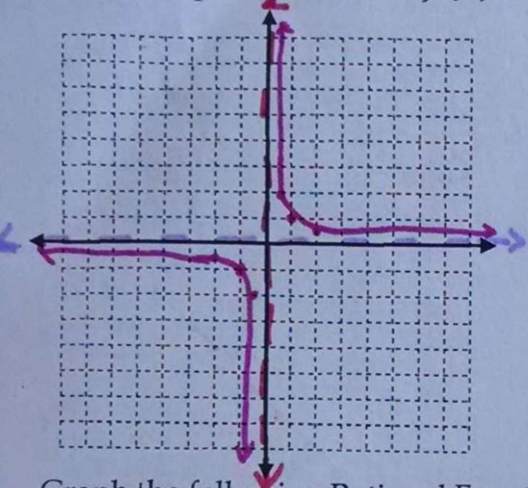
More with Rational Functions

$\frac{1}{0}$ = undef

Graph the parent function $f(x) = \frac{1}{x}$ and fill in characteristics.

$x=0$

deg N < deg D
↳ y=0



x	y
-2	-1/2
-1	-1
-0.5	-2
0	undefined
0.5	2
1	1
2	1/2

Domain: ℝ, $x \neq 0$

Range: ℝ, $y \neq 0$

Vertical Asymptote: $x=0$

Horizontal Asymptote: $y=0$

y-intercept: NONE

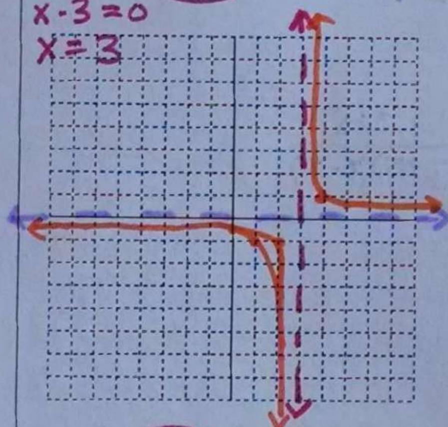
Zero(s) or x-intercept(s): None

Graph the following Rational Functions and describe the transformation from the parent function.

a. $g(x) = \frac{1}{x-3}$

Deg N < Deg D
↳ y=0

$x-3=0$
$x=3$



Domain: ℝ, $x \neq 3$

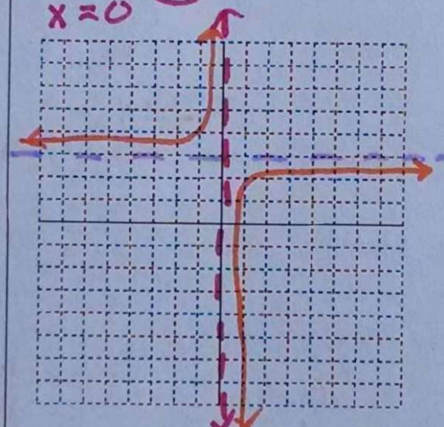
Range: ℝ, $y \neq 0$

Vertical Asymptote: $x=3$

Horiz. Asymptote: $y=0$

b. $h(x) = \frac{-1}{x} + 3$

$x=0$



Domain: ℝ, $x \neq 0$

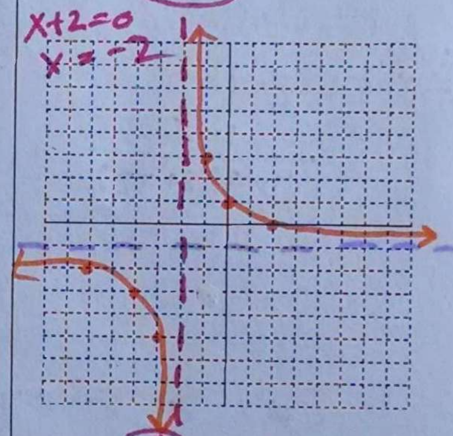
Range: ℝ, $y \neq 3$

Vertical Asymptote: $x=0$

Horiz. Asymptote: $y=3$

c. $g(x) = \frac{4}{x+2} - 1$

$x+2=0$
$x=-2$



Domain: ℝ, $x \neq -2$

Range: ℝ, $y \neq -1$

Vertical Asymptote: $x=-2$

Horiz. Asymptote: $y=-1$

Vertical stretch or compression

a<0 → Reflection across x-axis

$$f(x) = \frac{a}{x-h} + k$$

vertical Translation (Horizontal Asymptote)

Horizontal Translation (Vertical Asymptote)

Rational Functions

For a rational function of the form $f(x) = \frac{a}{x-h} + k$,

- the graph is a **hyperbola**
- there is a vertical asymptote at the line $x=h$, and the domain is ℝ, $x \neq h$
- there is a horizontal asymptote at the line $y=k$, and the range is ℝ, $y \neq k$

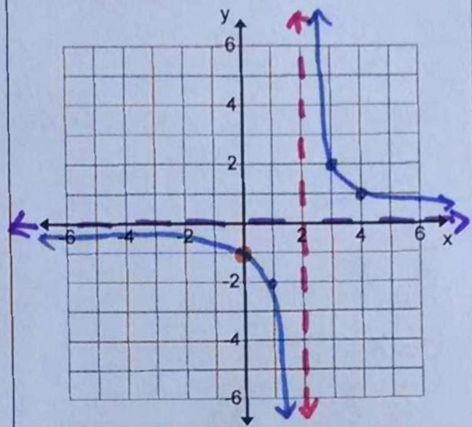
A Discontinuous function is a function whose graph has one or more gaps or breaks. Many rational functions are discontinuous functions.

A Continuous function is a function whose graph has no gaps or breaks. The functions you have studied before this, including linear, quadratic, polynomial, exponential, and logarithmic functions, are continuous functions.

TYPES OF DISCONTINUITY

INFINITE (asymptotes)

$f(x) = \frac{2}{(x-2)}$



VA: $x-2=0 \rightarrow x=2$

H.A: deg N < Deg D $\rightarrow y=0$

y-int: $2/(0-2) = (0,-1)$

x-int: None

POINT (hole)

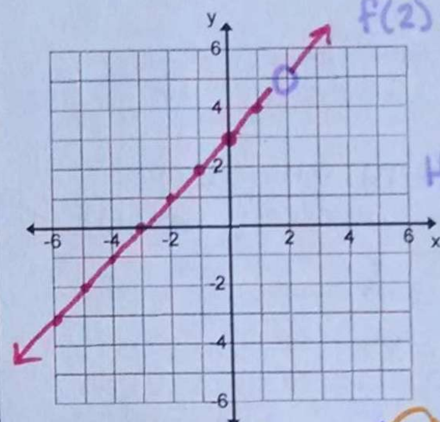
$f(x) = \frac{\cancel{(x-2)}(x+3)}{\cancel{(x-2)}}$ $f(x) = x+3$, $x \neq 2$

$f(2) = 2+3$

$= 5$

$(2, 5)$

Hole



Domain: $\mathbb{R}$, $x \neq 2$

Range: $\mathbb{R}$, $x \neq 5$

If a rational function has the same factor $x-b$ in both the numerator and denominator, then there is a hole in the grade at the point where $x = b$, unless the line $x = b$ is a vertical asymptote.

FYI: The calculator may not always help you identify the holes, so know how to FACTOR!

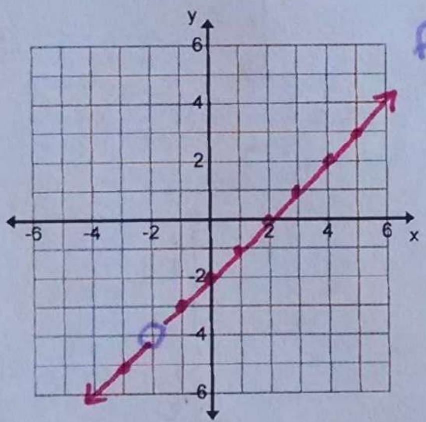
6. $f(x) = \frac{x^2-4}{x+2}$ $\frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$ $f(x) = x-2$, $x \neq -2$

$f(-2) = -2-2$

$= -4$

$(-2, -4)$

Hole@



Hole at x = -2 Coordinate of Hole (-2 , -4)

Domain: $\mathbb{R}$, $x \neq -2$ Range: $\mathbb{R}$, $y \neq -4$

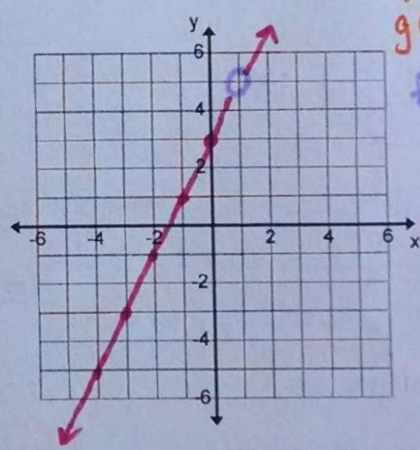
7. $g(x) = \frac{2x^2+x-3}{x-1}$ $\frac{(2x+3)\cancel{(x-1)}}{\cancel{x-1}}$

$g(x) = 2x+3$, $x \neq 1$

$f(1) = 2(1)+3$

$= 5$

$(1, 5)$



Hole at x = 1 Coordinate of Hole (1 , 5)

Domain: $\mathbb{R}$, $x \neq 1$ Range: $\mathbb{R}$, $y \neq 5$

GLUE HERE