

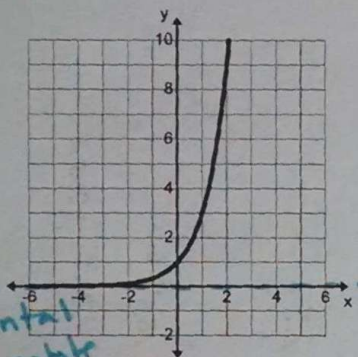


# Exponential

## Growth

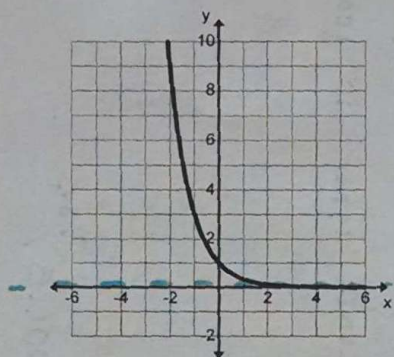
&

## Decay



$$f(x) = ab^x$$

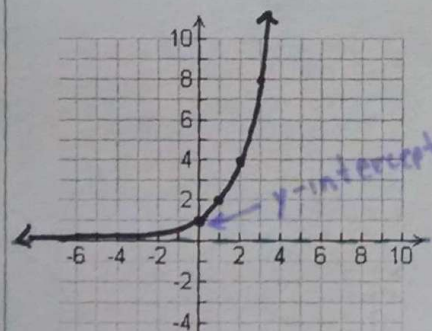
initial amount



$$f(x) = ab^x$$

initial amount

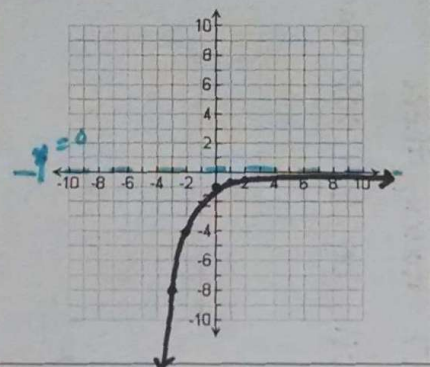
$$y = 2^x$$



D:  $\mathbb{R}$

$$R: y \geq 0$$

$$y = -\left(\frac{1}{2}\right)^x$$



D:  $\mathbb{R}$

$$R: y \leq 0$$

$a < 0 \rightarrow$  Reflect across x-axis

$0 < b < 1 \rightarrow$  decreasing

You can model a growth or decay by a constant percent increase or decrease with the following formula:

$$\text{Amount at a time, } t \rightarrow A(t) = a(1 \pm r)^t$$

$\swarrow$  initial amount
 $\swarrow$  time
 $\swarrow$  Rate (decimal)

### Growth

When a function increases by a constant rate, such as 7%, this is the same as multiplying by  $100\% + 7\%$ , or 107%. As a decimal

$$1 + 0.07, \text{ or } 1.07$$

### Decay

When a function decreases by a constant rate, such as 7%, this is the same as multiplying by  $100\% - 7\%$ , or 93%. As a decimal

$$1 - 0.07, \text{ or } 0.93$$

Tony purchased a rare 1959 Gibson Les Paul guitar in 2010 for \$12,000. Experts estimate that its value will increase by 14% per year. How much will the guitar be worth in 2017? When will the value of the guitar be \$60,000?

$$a = 12000 \quad r = 14\% \rightarrow .14$$

$$y = 12000(1 + .14)^x$$

$$= 30,027.23$$

The value of a truck bought new for \$28,000 decreases by 9.5% each year. What will the truck be worth in 5 years? When will the truck value fall to \$5000?

$$a = 28000 \quad r = 9.5\% \Rightarrow .095$$

$$y = 28000(1 - .095)^x$$

$$y = 16,998.12$$

A Half - Life is a specific kind of decay that represents the time it takes for the count rate to fall to half of its original reading.

$$A(n) = a(.5)^n$$

$\swarrow$  initial amount
 $\swarrow$  # of half-lives (total time / half-life)

$\swarrow$  Amount Remaining

An alien radioactive isotope has a half-life of 238 years. If you start with a sample of 8kg, how much will be left in 100 years?

$$y = 8(.5)^{100/238} \Rightarrow 5.97\text{kg}$$

How much will be left after 500 years?

$$y = 8(.5)^{500/238} \Rightarrow 1.86\text{kg}$$

The half - life of Zion - 71 is 48.5 seconds. If you had 100g, how much would be left after 3 minutes?

$$y = 100(.5)^{180/48.5}$$

$$7.63\text{g}$$

$$15(1 + .25)^x$$

$$15(1.25)^x$$

↑ 25%

$$15(.80)^x$$

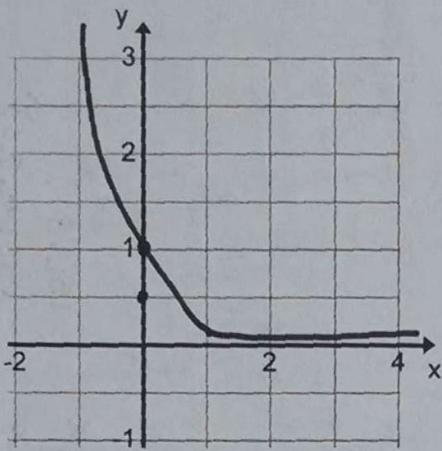
$$15(1 - .20)^x$$

↓ 20%

$$f(x) = 0.25^x \quad \text{Growth or Decay}$$

$$a = \underline{1} \quad b = \underline{.25}$$

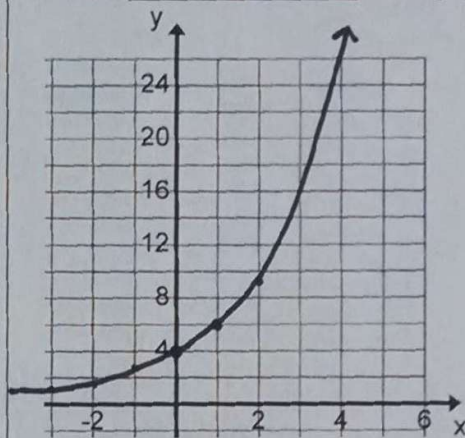
$$\text{Domain: } \underline{\mathbb{R}} \quad \text{Range: } \underline{y \geq 0}$$



$$y = 4\left(\frac{3}{2}\right)^x \quad \text{Growth or Decay}$$

$$a = \underline{4} \quad b = \underline{3/2}$$

$$\text{Domain } \underline{\mathbb{R}} \quad \text{Range } \underline{y \geq 0}$$



An acidophilus culture containing 150 bacteria doubles in population every hour. Predict the number of bacteria after 12 hours.

$$y = 150(2)^x \quad \text{or} \quad y = 150(1 + 1)^x$$

after 12 hours, there will be 614,000 bacteria

The population of Campbell increases by 5% every year. If there are currently 83,000 people living there, what will the population be in 10 years?

$$y = 83000(1 + .05)^x$$

There will be 135,198 residents after 10 years

A culture of yeast increases in size by about 1.5 times every 20 minutes. If the size of the culture is 2, find its size in 2 hours.

$$y = 2(1.5)^{\frac{120}{20}}$$

$$y = 2(1.5)^6$$

$$22.78$$

The half-life of radon is approximately 3.5 days. Determine how much of a 100 g sample will remain after 1 week.

$$y = 100(.5)^{7/3.5}$$

$$y = 100(.5)^2$$

$$= 25 \text{ g}$$

How long until less than 12 grams remains?

$$y = 100(.5)^{x/3.5}$$

$$\approx 10.3 \text{ years}$$