

$$f(x) = x^2, g(x) = 2x - 3, h(x) = \sqrt{x+1}$$

$g(f(x))$

$$\begin{aligned} f(g(x)) &= x^2 \\ f(x) = x^2, g(x) = 2x-3 & \\ (2x-3)^2 & \\ (2x-3)(2x-3) & \\ 4x^2 - 12x + 9 & \end{aligned}$$

$(g \circ f)(x)$

$$\begin{aligned} g(x) = 2x-3, f(x) = x^2 & \\ 2(x^2) - 3 & \\ 2x^2 - 3 & \end{aligned}$$

$(f \circ h)(x)$

$$\begin{aligned} f(x) = x^2 & \\ h(x) = \sqrt{x+1} & \\ (\cancel{x+1})^2 & \\ x+1 & \end{aligned}$$

$$\begin{aligned} h(g(x)) & \\ h(x) = \sqrt{x+1} & \\ g(x) = 2x-3 & \\ \sqrt{(2x-3)+1} & \\ \sqrt{2x-2} & \end{aligned}$$

$$\begin{aligned} g(h(x)) & \\ g(x) = 2x-3 & \\ h(x) = \sqrt{x+1} & \\ 2(\sqrt{x+1}) - 3 & \\ 2\sqrt{x+1} - 3 & \end{aligned}$$

$$\begin{aligned} (h \circ f)(x) & \\ h(x) = \sqrt{x+1} & \\ f(x) = x^2 & \\ \sqrt{x^2+1} & \end{aligned}$$

Operations with Functions

Notation for Function Operations

Operation	Notation
Addition	$(f+g)(x) = f(x)+g(x)$
Subtraction	$(f-g)(x) = f(x)-g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

Example 1: Given $f(x) = 2x^2 + 4x - 6$ and $g(x) = 2x - 2$, find each value.

a. $(f+g)(2)$	b. $(f-g)(-4)$	c. $(fg)(-1)$
$f(2) + g(2)$ $f(2) = 2(2)^2 + 4(2) - 6$ $= 16 + 8 - 6$ $= 18$	$f(-4) - g(-4)$ $f(-4) = 2(-4)^2 + 4(-4) - 6$ $= 32 - 16 - 6$ $= 10$	$f(-1) \cdot g(-1)$ $f(-1) = 2(-1)^2 + 4(-1) - 6$ $= 2 - 4 - 6$ $= -8$
	$10 - (-10)$ $= 20$	$(-8) \cdot (-4)$ $= 32$

Example 2: Given $f(x) = 2x^2 + 4x - 6$ and $g(x) = 2x - 2$, find expression.

a. $(f+g)(x)$	b. $(f-g)(x)$
$f(x) + g(x)$ $(2x^2 + 4x - 6) + (2x - 2)$ $2x^2 + 4x - 6 + 2x - 2$ $2x^2 + 6x - 8$	$f(x) - g(x)$ $(2x^2 + 4x - 6) - (2x - 2)$ $2x^2 + 4x - 6 - 2x + 2$ $2x^2 + 2x - 4$

c. $(fg)(x)$	d. $\left(\frac{f}{g}\right)(x)$
$g(x) \cdot f(x)$ $(2x-2)(2x^2 + 4x - 6)$ $4x^3 + 8x^2 - 12x - 4x^2 - 8x + 12$ $4x^3 + 4x^2 - 20x + 12$	$\frac{f(x)}{g(x)}$ $\frac{2x^2 + 4x - 6}{2x - 2}$ $\frac{2(x^2 + 2x - 3)}{2(x-1)}$

$2x^2 + 4x - 6$
$2x \quad 4x^3 \quad 8x \quad -12x$
$-2 \quad -4x^2 \quad -8x \quad +12$

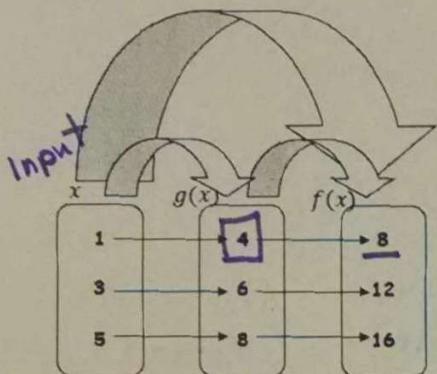
$$4x^3 + 4x^2 - 20x + 12$$

$$\frac{2(x-1)(x+3)}{2(x-1)}$$

$$x+3$$

Composition of Functions

The composition of functions $f(x)$ and $g(x)$ is notated $(f \circ g)(x) = f(g(x))$.



$f(g(1))$

To find $(f \circ g)(1)$, first find $g(1)$.

Then use 4 as the input into f :

$$\text{So } (f \circ g)(1) = \underline{\hspace{2cm}} = 8$$

The order of function operations is the same as the order of operations for numbers and expressions (parentheses FIRST)

$f(g(3))$, evaluate $g(3)$ first and then substitute the result into f .

$g(f(3))$, evaluate $f(3)$ first and then substitute the result into g .

Example 3: Given $f(x) = 3x + 1$ and $g(x) = x^3$, find each value.

a. $f(g(2))$

$$g(2) = (2)^3$$

$$= 8$$

$$f(8) = 3(8) + 1$$

$$f(g(2)) = \boxed{25}$$

b. $(g \circ f)(2)$

$$g(f(2))$$

$$f(2) = 3(2) + 1$$

$$= 7$$

$$g(7) = (7)^3$$

$$g(f(2)) = \boxed{343}$$

c. $f(g(-3))$

$$g(-3) = (-3)^3$$

$$= -27$$

$$f(-27) = 3(-27) + 1$$

$$f(g(-3)) = \boxed{-80}$$

Example 4: Given $f(x) = 5x + 2$ and $g(x) = \frac{2}{x-1}$, write each composite function

a. $f(g(x))$

b. $(g \circ f)(x)$

Example 5: Use the tables to find each value.

f(x)

x	1	2	3	4	5
$f(x)$	1	0	1	4	9

x	3	4	5	6	7
$g(x)$	0	2	4	6	8

g(x)

a. $(f+g)(4)$

$$f(4) + g(4)$$

$$4 + 2$$

$$\boxed{6}$$

b. $\left(\frac{g}{f}\right)(5)$

$$\frac{g(5)}{f(5)}$$

$$\frac{4}{9}$$

c. $(g \circ f)(4)$

$$g(f(4))$$

$$f(4) = 4$$

$$g(4) = \boxed{2}$$

d. $(g-f)(5)$

$$g(5) - f(5)$$

$$4 - 9$$

$$\boxed{-5}$$

e. $(fg)(3)$

$$f(3) \cdot g(3)$$

$$1 \cdot 3$$

$$\boxed{0}$$

f. $(f \circ g)(4)$

$$f(g(4))$$

$$f(2)$$

$$\boxed{0}$$