Pre-Geometry Lesson #2: Advanced Pythagorean Theorem

Using Pythagorean Theorem With Radicals

Assume with the examples below that ΔABC is a right triangle and that angle C is the right angle. Find the length of the missing side. (No decimal approximations, leave answers as simplified radicals).....

$a = 11, b = ?, c = 15$

$a^2+b^2=c^2$

$11^2+b^2=15^2$

$121+b^2=225$

$\sqrt{b^2}=\sqrt{104}$

$\sqrt{4}\sqrt{26}$

$\boxed{b=2\sqrt{26}}$

$a = 10, b = 6\sqrt{3}, c = ?$

$a^2+b^2=c^2$

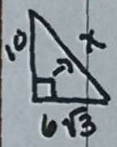
$10^2+(6\sqrt{3})^2=c^2$

$100+108=c^2$

$\sqrt{208}=\sqrt{c^2}$

$\sqrt{16}\sqrt{13}=c$

$\boxed{4\sqrt{13}=c}$



$a = ?, b = 6, c = 3\sqrt{6}$

$a^2+6^2=(3\sqrt{6})^2$

$a^2+36=54$

$\sqrt{a^2}=\sqrt{18}$

$a=\sqrt{9}\sqrt{2}$

$\boxed{a=3\sqrt{2}}$

$a = 5\sqrt{5}, b = 2\sqrt{5}, c = ?$

$(5\sqrt{5})^2+(2\sqrt{5})^2=c^2$

$125+20=c^2$

$\sqrt{145}=\sqrt{c^2}$

$\boxed{c=\sqrt{145}}$

Determining Whether or Not Three Numbers can represent the Side Lengths of a Right Triangle?

Square all three numbers individually and get a value for each. Then, calculate if the two smaller values add up to the larger value. If not, then the three numbers cannot represent the lengths of the sides of a right triangle. If so, then they can.

Here's how.....

$11, 7\sqrt{2}, \text{and } 4$

11^2 $(7\sqrt{2})^2$ 4^2

↓ ↓ ↓

121 98 16

$98 + 16 \neq 121$

Not a Right Triangle

$2\sqrt{19}, 6, \text{and } 2\sqrt{10}$

$(2\sqrt{19})^2$ 6^2 $(2\sqrt{10})^2$

↓ ↓ ↓

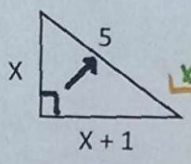
76 36 40

$36 + 40 = 76$ ✓

∴ the sides form a Right Triangle

Using Pythagorean Theorem With Polynomial Side Lengths

Many times you will be given the side lengths of a right triangle in terms of "x" and be asked to use the Pythagorean Theorem to find "x" and the side lengths. Here's is the process:



$x^2+(x+1)^2=5^2$

$x^2+x^2+2x+1=25$

$2x^2+2x+1=25$

$2x^2+2x-24=0$

$\frac{2}{2}(x^2+x-12)=\frac{0}{2}$

$x^2+x-12=0$

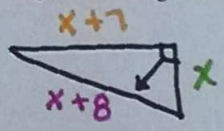
$(x+4)(x-3)=0$

$x+4=0$ $x-3=0$

~~$x=-4$~~ $\boxed{x=3}$

3, 4, 5

In a right triangle, one leg is seven inches more than the other leg and the hypotenuse is one inch more than the longest leg. Find the length of the legs and the hypotenuse.



$x^2+(x+7)^2=(x+8)^2$

$x^2+x^2+14x+49=x^2+16x+64$

$2x^2+14x+49=x^2+16x+64$

$-x^2-16x-64$ $-x^2-16x-64$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x-5=0$ $x+3=0$

$\boxed{x=5}$ ~~$x=-3$~~

5, 12, 13